MATHEMATICS

Part - 2

Standard
VIII

Government of Kerala
Department of General Education

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2025

THE NATIONAL ANTHEM

Jana-gana-mana adhinayaka, jaya he
Bharatha-bhagya-vidhata
Punjab-Sindh-Gujarat-Maratha
Dravida-Utkala-Banga
Vindhya-Himachala-Yamuna-Ganga
Uchchala-Jaladhi-taranga
Tava subha name jage,
Tava subha asisa mage,
Gahe tava jaya gatha
Jana-gana-mangala-dayaka jaya he
Bharatha-bhagya-vidhata
Jaya he, jaya he, jaya he,
Jaya jaya jaya, jaya he.

PLEDGE

India is my country. All Indians are my brothers and sisters.

I love my country, and I am proud of its rich and varied heritage. I shall always strive to be worthy of it.

I shall give my parents, teachers and all elders, respect and treat everyone with courtesy.

To my country and my people, I pledge my devotion. In their well-being and prosperity alone, lies my happiness.

MATHEMATICS

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Dear children,

We have seen in earlier classes how natural numbers and fractions are used to denote various measures. We have also seen how different combinations of measurements lead to mathematical operations on pure numbers and how the general principles of such operations are written using algebra. We also recognised special properties of geometric shapes such as rectangles, triangles and circles.

This book continues these studies. The method of solving practical problems on measures and the numbers denoting them using algebra, starts here. This method of converting physical problems to mathematical problems and solving them using algebraic techniques is used in almost all sciences. Because of this, it is an important aspect of mathematics education at all levels.

We've seen in the textbooks for previous classes and also in the first part of the textbook for this class, many concepts and applications of various branches of mathematics, such as arithmetic which deals with numbers and their operations, algebra which provides a shorthand notation for these, geometry which studies various shapes and statistics which analyses numerical data. This part of the textbook will give more knowledge on all these.

The idea of ratios introduced in class 7 is presented here using algebra and its applications in geometry are discussed. The concept of decimal representation is developed into a different form here. The construction of triangles according to specifications, recognition of the relations between sides and angles, and the computation of areas are extended to quadrilaterals. New ways to represent numerical data as tables and pictures are also included.

We hope that this book will help you to understand the breadth of applications of mathematics and appreciate the beauty of its logic.

With love and regards,

Dr. Jayaprakash R.K.
Director
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THE CONSTITUTION OF INDIA

PREAMBLE

WE, THE PEOPLE OF INDIA, having solemnly resolved to constitute India into a [1]**[SOVEREIGN SOCIALIST SECULAR DEMOCRATIC REPUBLIC]** and to secure to all its citizens :

JUSTICE, social, economic and political;

LIBERTY of thought, expression, belief, faith and worship;

EQUALITY of status and of opportunity; and to promote among them all

FRATERNITY assuring the dignity of the individual and the [2][unity and integrity of the Nation];

IN OUR CONSTITUENT ASSEMBLY this twenty-sixth day of November, 1949 do **HEREBY ADOPT, ENACT AND GIVE TO OURSELVES THIS CONSTITUTION.**

---

1. Subs. by the Constitution (Forty-second Amendment) Act, 1976, Sec.2, for "Sovereign Democratic Republic" (w.e.f. 3.1.1977)
2. Subs. by the Constitution (Forty-second Amendment) Act, 1976, Sec.2, for "Unity of the Nation" (w.e.f. 3.1.1977)

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RATIO

Multiplicative comparisons

See this picture:

We can compare the lengths of the screws in many ways.

Using addition and subtraction:

- The length of the short screw is 8 millimetres less than the length of the long screw.
- The length of the long screw is 8 millimetres more than the length of the short screw.

We can also compare using multiplication and division:

- The length of the short screw is half the length of the long screw.
- The length of the long screw is twice the length of the short screw.

We have seen in Class 7 that the second kind of comparison can be said in a different manner:

- The ratio of the length of the short screw to the length of the long screw is 1 to 2.
- The ratio of the length of the long screw to the length of the short screw is 2 to 1.

(The section **Other measures** of the lesson **Ratio** in the Class 7 textbook).

In the same lesson, we have seen that the ratio 1 to 2 is written 1 : 2 and the ratio 2 to 1 is written 2 : 1

Now see these screws:

How do you state the relation between their lengths as a ratio?

For that, we will have to compute what fraction of the length of the long screw is the length of the short screw.

That is, what fraction of 12 is 8?

Remember doing such problems in Class 7? (The section **Times and parts** of the lesson **Reciprocals**).

1 is $\frac{1}{12}$ of 12; and 8 is 8 times 1.

Thus 8 is $\frac{8}{12}$ of 12.

We can remove common factors and write $\frac{8}{12}$ like this:

$$\frac{8}{12} = \frac{2 \times 4}{3 \times 4} = \frac{2}{3}$$

(The section **Numerator and denominator** of the lesson **One Fraction, Many Forms**).

So the length of the short screw is $\frac{2}{3}$ of the length of the long screw.

Thus the ratio of the length of the short screw to the length of the long screw is 2 : 3.

We have seen that not only lengths, but other measures also can be compared using ratios.

For example, if we consider a 15 litre-bucket and a 20 litre-bucket, the smaller bucket can contain only $\frac{15}{20}$ of what the large bucket can contain. Reducing the fraction to its lowest terms,

$$\frac{15}{20} = \frac{3}{4}$$

So, the ratio of the small bucket's capacity to the large bucket's capacity is 3 : 4.

(1) In each of the following pairs of measures, find the ratio of the smaller to the larger:

(i) 4 metres, 12metres (ii) 8 litres, 20 litres

(iii) 20 kilograms, 25 kilograms

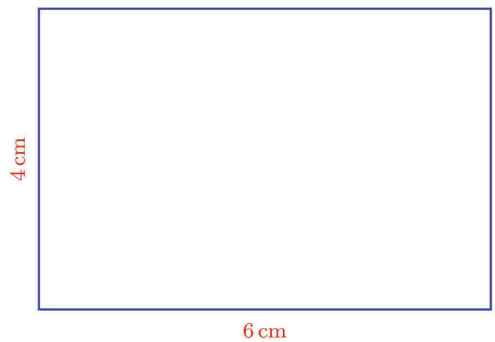
(2) A can contains 4 litres of water and another can, 14 litres.

(i) What is the ratio of the amount of water in the smaller can to the amount of water in the larger can?

(ii) If one more litre of water is poured into each can, what would this ratio become?

Algebra

See this rectangle:



What is the height to width ratio?

We can calculate it like this:

(i) The height is $\frac{4}{6}$ of the width.

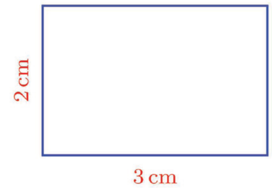
(ii) $\frac{4}{6}$ in lowest terms is $\frac{2}{3}$.

(iii) The height to width ratio is 2 : 3.

(We have seen in Class 7 that ratios are usually written using the smallest possible natural numbers).

Can you draw another rectangle with the height to width ratio 2 : 3.

An easy way to do this shown alongside.



Another rectangle?

The first rectangle we saw was drawn with twice 2 centimetres and 3 centimetres as height and width; what if we take three times these lengths as height and width?

In such a rectangle also, would the height to width ratio be 2 : 3?

Let's think about this as in the first problem:

(i) The height is $\frac{6}{9}$ of the width

(ii) $\frac{6}{9} = \frac{2 \times 3}{3 \times 3} = \frac{2}{3}$

(iii) The height is $\frac{2}{3}$ of the width

(iv) The height to width ratio is 2 : 3

What if we take $\frac{3}{4}$ of the width and height?

$$\frac{3}{4} \times 2 = 1\frac{1}{2}$$

$$\frac{3}{4} \times 3 = 2\frac{1}{4}$$

The height becomes $1\frac{1}{2}$ centimetres and the width $2\frac{1}{4}$ centimetres. Is the height to width ratio still 2 : 3?

That is, is $1\frac{1}{2}$ centimetres, $\frac{2}{3}$ of $2\frac{1}{4}$ centimetres?

We can check this using the multiplications done above:

$$\begin{aligned} \frac{2}{3} \times 2\frac{1}{4} &= \frac{2}{3} \times \left(\frac{3}{4} \times 3\right) \\ &= \frac{3}{4} \times \left(\frac{2}{3} \times 3\right) \\ &= \frac{3}{4} \times 2 \\ &= 1\frac{1}{2} \end{aligned}$$

So in this rectangle also, the height to width ratio is 2 : 3.

We can see like this, that the ratio will not change even if we take some other fraction of the lengths instead of $\frac{3}{4}$.

In these examples, we multiplied 2 centimetres and 3 centimetres by the same number to change them. So, what can we say in general?

> If the height and width are 2 centimetres and 3 centimetres multiplied by the same number, then its height to width ratio is 2 : 3.

On the other hand, for any rectangle with the height to width ratio 2 : 3, are the height and width 2 centimetres and 3 centimetres multiplied by the same number?

To check this let's denote the height and width of a rectangle with the height to width ratio 2 : 3 as h centimetres and w centimetres.

Since the height to width ratio is 2 : 3, the height is $\frac{2}{3}$ of the width; that is

$$h = \frac{2}{3}w$$

$\frac{2}{3}$ is 2 times $\frac{1}{3}$, right? (The section **Multifold multiplication** of the lesson **Fractions** in the Class 7 textbook).

So we can write $\frac{2}{3}w$ like this

$$\frac{2}{3}w = 2 \times \frac{1}{3}w$$

Now if we denote the number $\frac{1}{3}w$ by k, then

$$h = 2k$$

Do we also get $w = 3k$?

$$3k = 3 \times \frac{1}{3}w = w$$

Thus the height and width of the rectangle are 2 centimetres and 3 centimetres multiplied by the same number k.

In any rectangle with the height to width ratio 2 : 3, the height and the width are 2 centimetres and 3 centimetres multiplied by the same number.

It is not difficult to see that the two results above hold whatever be the natural numbers we choose instead of 2 and 3.

Not only that, these are true, for any two lengths instead of the height and width of a rectangle and what is more, for any other pair of measures instead of lengths.

For example, if we consider a 4-litre can and a 10-litre can, what fraction of the capacity of the large can is the capacity of the small can?

$$\frac{4}{10} = \frac{2}{5}$$

So, the ratio of the capacity of the small can to the capacity of the large can is 2 : 5.

Now in any two cans with capacities 2 litres and 5 litres multiplied by the same number, the ratio of the capacity of the small can to the capacity of the large can is 2 : 5; on the other hand, for any two cans with the ratio of their capacities 2 : 5, their actual capacities are 2 litres and 5 litres multiplied by the same number.

Thus we can say this in general

If two measures are in the ratio $m : n$, the ratio of the measures got by multiplying m and n by any non-zero number would also be $m : n$; on the other hand, if the ratio of two measures is $m : n$, then their actual measures are m and n multiplied by the same number

Let's redo a problem from Class 7 in a different way using this (The section **Division problem** of the lesson **Ratio**).

The school needs a vegetable garden. A rectangular plot is to be roped off for this. The length of the rope is 32 metres. They decided to have width and length in the ratio 3 : 5. What should be the width and the length?

Since the width to length ratio is 3 : 5, width and length must be 3 metres and 5 metres multiplied by the same number, as we have just noted. To calculate the actual width and length, we need only find this number. For that, let's use algebra.

If we denote this number by x, we have

$$\text{Width} = 3x \text{ metres.}$$

$$\text{Length} = 5x \text{ metres.}$$

Now to find x, we use another fact given in the problem. What do we get from the information that the length of the rope is 32 metres?

(i) The perimeter of the rectangle is 32 metres.

(ii) The sum of width and length is half of 32, which is 16.

The width and length are $3x$ metres and $5x$ metres; and their sum is $3x + 5x = 8x$ metres. We have seen above that the sum of width and length is 16 metres. Thus

$$8x = 16$$

From this we can get x:

$$x = 2$$

(Recall the lesson **Solutions of Equations**).

Thus we find,

$$\text{Width} = 3x = 3 \times 2 = 6 \text{ metres.}$$

$$\text{Length} = 5x = 5 \times 2 = 10 \text{ metres.}$$

Let's look at another problem:

The width and length of a rectangle are in the ratio 4 : 5 and its area is 320 square metres. What are the width and length?

As in the first problem, we can take the width and length as $4x$ metres and $5x$ metres.

Then the area is

$$4x \times 5x = 20x^2 \text{ square metres.}$$

The area is said to be 320 square metres. So,

$$20x^2 = 320$$

From this we get

$$x^2 = 320 \div 20 = 16$$

This means the square of the number x is 16. So, what is x?

$$x = 4$$

Now can't we calculate the width and length?

$$\text{Width} = 4x = 4 \times 4 = 16 \text{ metres.}$$

$$\text{Length} = 5x = 5 \times 4 = 20 \text{ metres.}$$

(1) The ratio of the inner and outer angles of a regular polygon is 7 : 2.

(i) How much is each angle?

(ii) How many sides does the polygon have?

(2) There are four right triangles, all with ratio of perpendicular sides 3 : 4. One more information about each triangle is given below. Calculate the lengths of all three sides of each:

(i) The difference in lengths of the perpendicular sides is 24 metres.

(ii) The perimeter of the triangle is 24 metres.

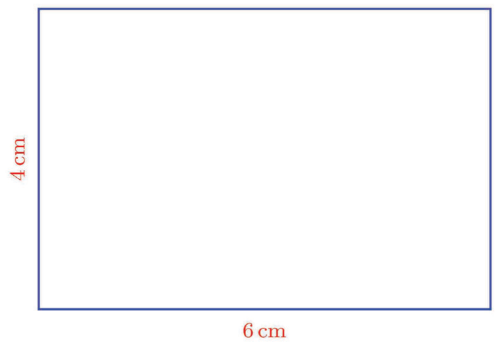
(iii) The area of the triangle is 24 square metres.

(iv) Hypotenuse 24 metres (Write the lengths in metres and centimetres).

(3) The sides of two squares are in the ratio 3 : 4. What is the ratio of their areas?

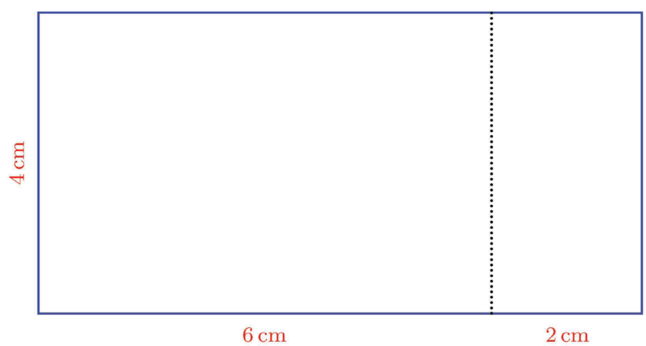
Changing ratios

What is the height to width ratio of this rectangle?



Since the height is 2×2 centimetres and width is 3×2 centimetres, the height to width ratio is $2 : 3$.

What if the rectangle is enlarged by increasing width alone by 2 centimetres?



Now the height to width ratio changes to $1 : 2$, right?

Here is a question in the other direction:

> The width to height ratio of a rectangle is $3 : 2$. When it was enlarged by increasing the width alone by 2 centimetres, the width to height ratio became $5 : 3$. What were the width and height of the original rectangle?

Since the width to height ratio of the original rectangle was $3 : 2$, we can take the actual width and height as $3x$ centimetres and $2x$ centimetres.

What about the enlarged rectangle?

When the width was increased by 2 centimetres, it became $3x + 2$ centimetres.

The height was not changed; it remained at $2x$ centimetres.

In the enlarged rectangle, the width to height ratio is said to be 5 : 3.

This means, the width is $\frac{5}{3}$ of height.

$$3x + 2 = \frac{5}{3} \times 2x$$

The right side of this equation can be written as

$$\frac{5}{3} \times 2x = \frac{1}{3} \times 10x$$

So the equation becomes

$$3x + 2 = \frac{1}{3} \times 10x$$

This means $\frac{1}{3}$ of the number $10x$ is the number $3x + 2$.

Looking at this the other way round, the number $10x$ is 3 times the number $3x + 2$.

That is,

$$10x = 3(3x + 2)$$

In this we can write $3(3x + 2)$ as

$$3(3x + 2) = (3 \times 3x) + (3 \times 2) = 9x + 6$$

and so the equation becomes

$$10x = 9x + 6$$

From this, can't we calculate x?

$$x = 6$$

Thus in the original rectangle,

$$\text{Width} = 3x = 3 \times 6 = 18 \text{ cm.}$$

$$\text{Height} = 2x = 2 \times 6 = 12 \text{ cm.}$$

Let's look at another problem:

Acid and water are mixed in the ratio 2 : 3 to make a mixture of 60 litres. To make this ratio 3 : 4, should more acid or water be added? How many litres?

Ratio in chemistry

In chemistry, substances are classified into elements and compounds, right?

In the eighteenth century, the chemist Joseph Proust discovered that in any compound, the mass of the elements in it are in a definite ratio.

For example, he discovered through experiments that in any sample of copper carbonate, the mass of copper is 5.3 times the mass of carbon and the mass of oxygen is 4 times the mass of carbon.

Perhaps what lead to the concept of an atom was the idea that if we imagine very small particles of elements, then we can express such comparisons in terms of natural numbers. This theory was first proposed by the scientist John Dalton in the nineteenth century. According to Dalton's theory, compounds are formed by very small parts of elements called atoms.

The number of atoms of different elements in any compound, is in a fixed ratio.

Acid and water are mixed in the ratio 2 : 3 means, the volume of acid is $\frac{2}{3}$ of the volume of water.

What does it mean to say that this ratio is to be made 3 : 4?

The volume of acid should be $\frac{3}{4}$ of the volume of water.

Which is larger, $\frac{2}{3}$ or $\frac{3}{4}$? (Recall the section **Large and small** of the lesson **Arithmetic of Parts** in the Class 6 textbook).

To check this, we must change both fractions to other forms with the same denominator.

$$\frac{2}{3} = \frac{2 \times 4}{3 \times 4} = \frac{8}{12}$$

$$\frac{3}{4} = \frac{3 \times 3}{4 \times 3} = \frac{9}{12}$$

Since $\frac{8}{12} < \frac{9}{12}$ we have $\frac{2}{3} < \frac{3}{4}$

So, we have to increase the volume of the acid.

To calculate how much acid is to be added, let's first calculate the actual volumes of acid and water in the current mixture. Since their ratio is 2 : 3 we can take them as

$$\text{Acid} = 2x \text{ litres}$$

$$\text{Water} = 3x \text{ litres}$$

Since the total volume is 60 litres, we have

$$5x = 60$$

From this, we get $x = 12$, so that

$$\text{Acid} = 24 \text{ litres}$$

$$\text{Water} = 36 \text{ litres}$$

Now let's denote by y, the volume of acid to be added to make the ratio 3 : 4. What will be the volumes then?

$$\text{Acid} = (24 + y) \text{ litres.}$$

$$\text{Water} = 36 \text{ litres.}$$

In this the volume of acid should be $\frac{3}{4}$ of the water. So,

$$24 + y = \frac{3}{4} \times 36 = 27$$

From this we get

$$y = 3$$

Theory and practice

Suppose in the acid problem, we don't first check whether acid or water is to be added and proceed to calculate the amount of water to be added?

If the volume of water to be added is denoted as x litres, then in the new mixture, acid would be 24 litres and the amount of water, $36 + x$ litres. The volume of water in this should be $\frac{4}{3}$ of the volume of acid. This gives the equation

$$36 + x = \frac{4}{3} \times 24 = 32$$

But from this we get $x = -4$. We can think of this as meaning the volume of water has to be reduced by 4 litres. Then the amount of acid is 24 litres and the amount of water is 32 litres; the ratio is 3 : 4, right?

But there is a problem. From a mixture of acid and water, how do we remove some water alone? So, the math is right, but the solution is impossible in practice.

This means 3 more litres of acid is to be added.

(1) The length to width ratio of a rectangle is 3 : 2. The rectangle is enlarged by increasing the length by half much. What is the length to width ratio of the new rectangle?

(2) The ratio of two angles is 1 : 2. When the smaller angle was increased by 6° and the larger angle was decreased by 6°, this ratio became 2 : 3. What were the original angles?

(3) In a mixture, acid and water are in the ratio 3 : 2. When 15 litres of acid was added to this, the ratio became 3 : 1. How many litres of acid and water are there in the current mixture?

(4) The height to width ratio of a rectangle of perimeter 54 centimetres is 4 : 5

(i) This ratio is to be made 2 : 3 by increasing the height or width. Which is to be increased? By how much?

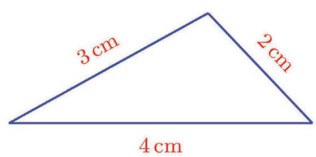
(ii) The ratio is to be made 2 : 3 by decreasing the height or width. Which is to be decreased? By how much?

Reversed ratio

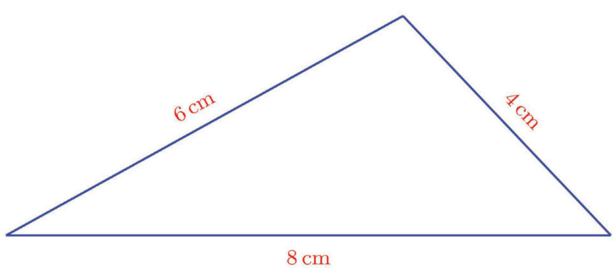
The length and width of a rectangle are 33 centimetres and 1 centimetre. Another rectangle has length and width 11 centimetres and 6 centimetres. What is the ratio of their perimeters? What about the ratio of the areas? Can you find other rectangles related like this?

Three measures

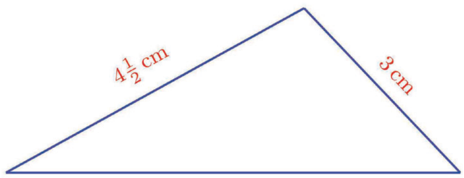
See this triangle:



We can draw another triangle with all its sides doubled:



If the sides of the first triangle are enlarged $1\frac{1}{2}$ times, then we get a triangle like this:



And if the sides are halved?

In each of the triangles above, the lengths of the sides are 4 centimetres, 3 centimetres and 2 centimetres multiplied by a definite number.

Sweet ratio

Do you know what all elements are there in sugar?

Carbon, hydrogen and oxygen in different quantities. One molecule of sugar is made up of 12 carbon atoms, 22 hydrogen atoms and 11 oxygen atoms. Thus the ratio of carbon, hydrogen and oxygen in sugar is 12 : 22 : 11. This is written in shorthand as $C_{12}H_{22}O_{11}$.

What happens when sugar is heated? Why?

This also we can state in terms of a ratio.

In all the four triangles, the sides are in the ratio 4 : 3 : 2.

We can state the relation between other types of three measures also as ratios.

For example the ratio of the capacities of a 15-litre vessel, a 30-litre vessel and a 10-litre vessel is 3 : 6 : 2.

The general definition can be stated like this:

Three measures which are got by multiplying the three numbers *l, m, n* by the same number are said to be in the ratio *l* : *m* : *n*

Let's look at a problem:

The ratio of the sides of a triangle is 3 : 5 : 7 and its perimeter is 45 centimetres. What are the lengths of its sides?

The lengths of the sides are 3 centimetres, 5 centimetres and 7 centimetres multiplied by the same number. Denoting this number as x, the perimeter is

$$3x + 5x + 7x = 15x \text{ cm}$$

This is said to be 45 centimetres. So

$$15x = 45$$

From this, we get $x = 3$ and using it, we can calculate the lengths of the sides as 9 centimetres, 15 centimetres and 21 centimetres.

Now look at this problem:

> In triangle *ABC*, the ratio of *AB* to *BC* is 3 : 4 and the ratio of *BC* to *CA* is 4 : 5. What is the ratio of the three sides together?

The actual lengths of *AB* and *BC* are 3 centimetres and 4 centimetres multiplied by the same number. Denoting this number as x, we have

$$AB = 3x \text{ cm}, \quad BC = 4x \text{ cm}$$

Again, the actual lengths of *BC* and *CA* are 4 centimetres and 5 centimetres multiplied by the same number.

In this, the length of *BC* is already seen to be 4 centimetres multiplied by x; so the length of *CA* must be 5 centimetres multiplied by the same number x. That is,

$$CA = 5x \text{ cm}$$

Since the lengths of *AB, BC* and *CA* are $3x$ centimetres, $4x$ centimetres and $5x$ centimetres, the ratio of these is 3 : 4 : 5

For measures other than lengths of the sides of a triangle also we can combine ratios like this. Thus we have the general rule:

> In three measures, if the ratio of the first to the second is $l : m$ and the ratio of the second to the third $m : n$, then the ratio of the three measures is $l : m : n$.

What if the ratios are like this?

> In triangle *ABC*, the ratio of *AB* to *BC* is 2 : 3 and the ratio of *BC* to *CA* is 4 : 5. What is the ratio of the three sides together?

The actual lengths of *AB* and BC are 2 centimetres and 3 centimetres multiplied by the same number. Denoting this number as x, we have

$$AB = 2x \text{ cm} \quad BC = 3x \text{ cm}$$

Again, the actual lengths of *BC* and *CA* are 4 centimetres and 5 centimetres multiplied by another number. Denoting this number as y, we have

$$BC = 4y \text{ cm} \quad CA = 5y \text{ cm}$$

The length of BC is $3x$ centimetres in the first equation and $4y$ centimetres in the second equation; so

$$4y = 3x$$

That is

$$y = \frac{3}{4}x$$

This gives

$$CA = 5y = 5 \times \frac{3}{4}x = \frac{15}{4}x$$

Now we can write all three lengths using x:

$$AB = 2x \text{ cm}$$

$$BC = 3x \text{ cm}$$

$$CA = \frac{15}{4}x \text{ cm}$$

We write ratios using only natural numbers. So, we rewrite these equations like this:

$$AB = 8 \times \frac{x}{4} \text{ cm}$$

$$BC = 12 \times \frac{x}{4} \text{ cm}$$

$$CA = 15 \times \frac{x}{4} \text{ cm}$$

Thus the lengths are 8 centimetres, 12 centimetres and 15 centimetres multiplied by the number $\frac{x}{4}$; so their ratio is 8 : 12 : 15.

Right triangles

What is the speciality of the triangle with sides 3 centimetres, 4 centimetres and 5 centimetres?

Since $3^2 + 4^2 = 5^2$, this is a right triangle.

If all these sides are doubled, do we still get a right triangle?

We also have $6^2 + 8^2 = 10^2$

This means we still get a right triangle.

What if we make all sides x times?

$$(3x)^2 + (4x)^2 = 9x^2 + 16x^2 = 25x^2 = (5x)^2$$

This means the triangle with sides $3x$, $4x$, $5x$ is right angled.

In short, any triangle with sides in the ratio 3 : 4 : 5 is a right triangle.

What about triangles with sides in the ratio 5 : 12 : 13?

(1) The angles of a triangle are in the ratio 1 : 3 : 5. What are the angles?

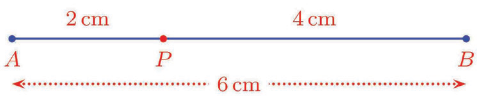
(2) The sides of a triangle are in the ratio 2 : 3 : 4 and the longest side is 20 centimetres longer than the shortest side. Calculate the length of all three sides.

(3) The ratio of the length, width and height of a rectangular box is 2 : 3 : 5 and its volume is 3750 cubic centimetres. Find the length, width and the height.

(4) A box contains beads of three colours. The ratio of the number of black beads to the number of white beads is 3 : 5 and the ratio of the number of white beads to the number of the red beads is 2 : 3. What is the ratio of the number of beads of all three colours?

Parts of a line

See this picture:



The point *P* divides the line *AB* into two parts, *AP* and *PB*.

What is the ratio of the lengths *AP* and *PB* ? Since *PB* is twice *AP*, the ratio of *AP* to *PB* is 1 : 2.

We can write it like this:

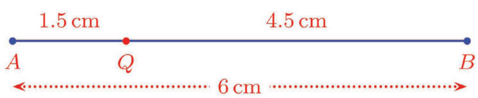
$$AP : PB = 1 : 2.$$

And we say it like this:

The point *P* divides the line *AB* in the ratio 1 : 2.

Thus any point on the line *AB* divides the line in a definite ratio.

For example, see this picture:



What is the ratio in which the point *Q* divides the line *AB* ?

4.5 is 3 times 1.5. Right ? So,

$$AQ : QB = 1:3$$

In this way each point of *AB* divides it in a definite ratio.

On the other hand, is there a point which divides AB in a specified ratio?

For example, which is the point dividing AB in the ratio 2 : 3?

We need to calculate the distance of the point from A.

That is, denoting that point as R, we must calculate the length of the part AR of AB.

Since $AR : RB = 2 : 3$, the lengths AR and RB must be 2 and 3 multiplied by the same number. Denoting this number as x, we must have

$$AR = 2x \text{ cm} \qquad RB = 3x \text{ cm}$$

Here we also have

$$AR + RB = AB = 6 \text{ cm}$$

This means we must have

$$5x = 6$$

This gives

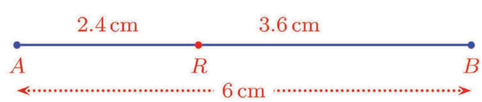
$$x = \frac{6}{5}$$

and so

$$AR = \frac{12}{5} \text{ cm} \quad RB = \frac{18}{5} \text{ cm}$$

It is more convenient here to use decimal forms

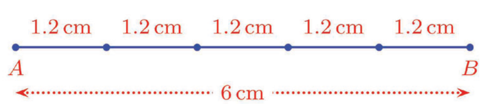
$$AR = \frac{24}{10} = 2.4 \text{ cm}, \quad RB = \frac{36}{10} = 3.6 \text{ cm}$$



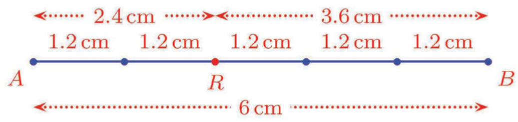
These computations can be seen geometrically also. How did we get x here?

$$x = \frac{6}{5} = \frac{12}{10} = 1.2$$

This means, if we divide 6 centimetres into 5 equal parts, then the length of each part is $x = 1.2$ centimetres.

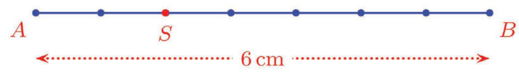


In this the first 2 parts together make *AR*, and the remaining 3 parts together make *RB*.



So where is the position of the point which divides *AB* in the ratio 2 : 5?

We have to divide *AB* into 2 + 5 = 7 equal parts and take the first 2 parts combined as *AS* and the remaining 5 parts combined as *SB*.



We cannot mark $\frac{6}{7}$ centimetres using a scale. We will see in Class 9, a method to divide a line in any specified ratio (the section **Unequal division** in the lesson **Parallel lines**).

So, what can we say in general about the point dividing a line in the ratio *m* : *n*?

The point dividing a line in the ratio *m* : *n* is that point which separates *m* + *n* equal parts of the line into the first *m* parts and the remaining *n* parts

For example, whatever be the length of a line, the point dividing it in the ratio 2 : 5 can be shown like this:

We can see another thing here:

- *AP* is 2 of the 7 equal parts of *AB* combined.
- *PB* is 5 of the 7 equal parts of *AB* combined.

That is,

- *AP* is $\frac{2}{7}$ of *AB*.
- *PB* is $\frac{5}{7}$ of *AB*.

What can we say in general?

If a line is divided in the ratio $m : n$, then the two parts are $\frac{m}{m+n}$ and $\frac{n}{m+n}$ of the whole line

(1) In each of the problems below, the length of a line and the ratio in which a point divides the line are specified. Draw each line and mark the point:

(i) $AB = 14$ cm $\quad AP : PB = 3 : 4$

(ii) $AB = 12$ cm $\quad AP : PB = 3 : 5$

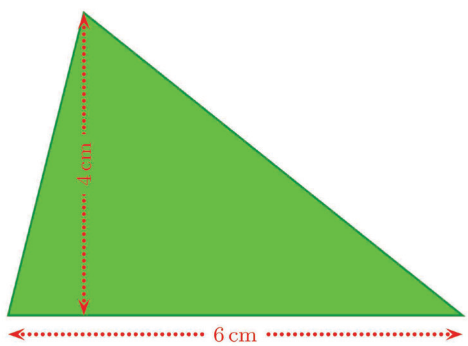
(iii) $AB = 7$ cm $\quad AP : PB = 2 : 3$

(2) A point divides a 9 centimetres long line in the ratio 2 : 3. Calculate the length of each part.

(3) Of the two parts got when a line is divided in the ratio 3 : 5, the longer part is 6 centimetres long. What is the length of the whole line?

Parts of a triangle

See this triangle:

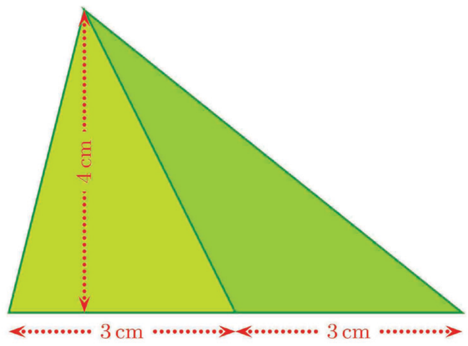


What is its area?

The length of one side is 6 centimetres and the height from it to the opposite vertex is 4 centimetres; so the area is

$$\frac{1}{2} \times 6 \times 4 = 12 \text{ sq.cm}$$

Now look at this picture:



The top vertex and the midpoint of the opposite side of the triangle we saw first are joined.

This line divides the triangle into two smaller triangles. What are the areas of these?

The bottom side of both have the same length. What about the height to the top vertex?

So, the area of each triangle is

$$\frac{1}{2} \times 3 \times 4 = 6 \text{ square centimetres}$$

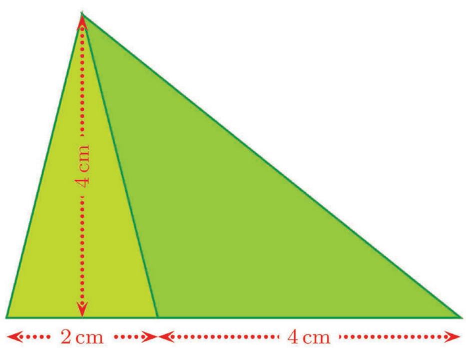
In other triangles with different measures also, the smaller triangles got like this have the same area, right?

What can we say in general?

> In any triangle, the line joining a vertex with the midpoint of the opposite side divides the triangle into two triangles of equal area.

Now suppose the top vertex is joined to some other point on the opposite side, instead of the midpoint?

For example, see this picture:



What are the areas of the smaller triangles?

Area of the left triangle is $\frac{1}{2} \times 2 \times 4 = 4$ sq.cm.

Area of the right triangle is $\frac{1}{2} \times 4 \times 4 = 8$ sq.cm.

This means the larger of the two triangles has twice the area of the smaller.

The bottom side is divided in the same way, isn't it? The longer piece has twice the length of the shorter.

How do you say it in terms of ratios?

The bottom side is divided in the ratio 1 : 2; the area of the triangle is also divided in the same ratio.

Does this hold for other ratios?

Let's think like this:

(i) A line from a vertex of a triangle divides the opposite side into two pieces; it divides the triangle into two triangles.

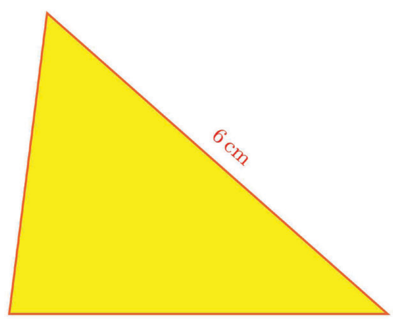
(ii) The areas of the triangles are products of the lengths of the pieces of the side by half the height of the triangle.

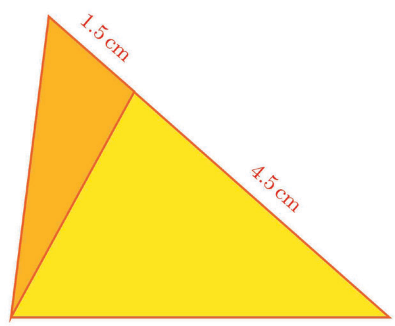
(iii) So the areas of the two triangles are in the same ratio as the lengths of the pieces of the side.

Summarizing, we have this:

A line joining a vertex of a triangle to the opposite side divides the length of the side and the area of the triangle in the same ratio

For example, look at these pictures:



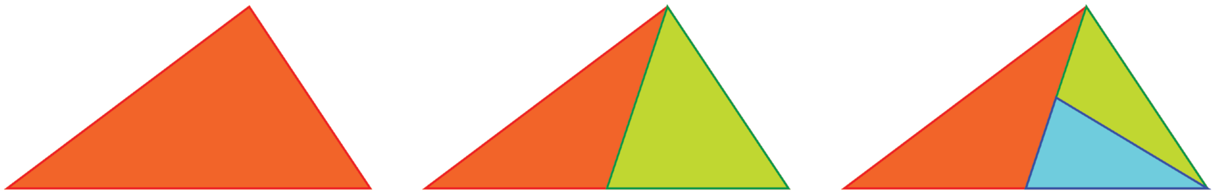


What is the ratio in which the line from the left vertex of the triangle divided the right side?

4.5 is 3 times 1.5, so that the short and long parts of the side are in the ratio 1 : 3.

The ratio of the areas of the small triangles in the picture on the right is also 1 : 3; that is the area of the larger of these two triangles is 3 times the area of the smaller.

Now see these pictures:



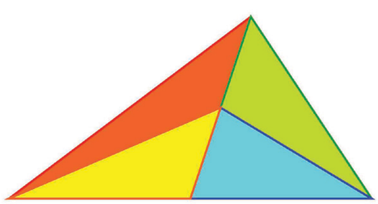
The top vertex and the midpoint of the opposite side of the triangle in the first are joined in the second picture.

The right vertex of the green triangle in the second picture is joined to the midpoint of its left side in the third picture.

What fractions of the area of original large triangle are the areas of the red, green, and blue triangles in the third picture?

Can you see that areas of the green and blue triangles are each a quarter of the area of the large triangle?

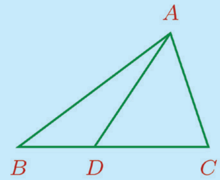
Now what if we join the left vertex of the red triangle to the midpoint of its right side also?



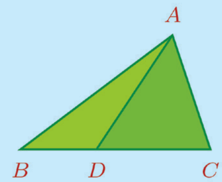
What fraction of the area of the large triangle is the area of each of the four small triangles?

Area relations

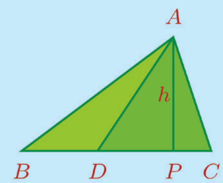
See this triangle:



What is the relation between the areas of the triangles ABD and ACD?



Draw the perpendicular from A to BC.



If the length of this perpendicular is denoted by h, then

$$\text{Area of } \Delta ABD = \frac{1}{2} h \times BD$$

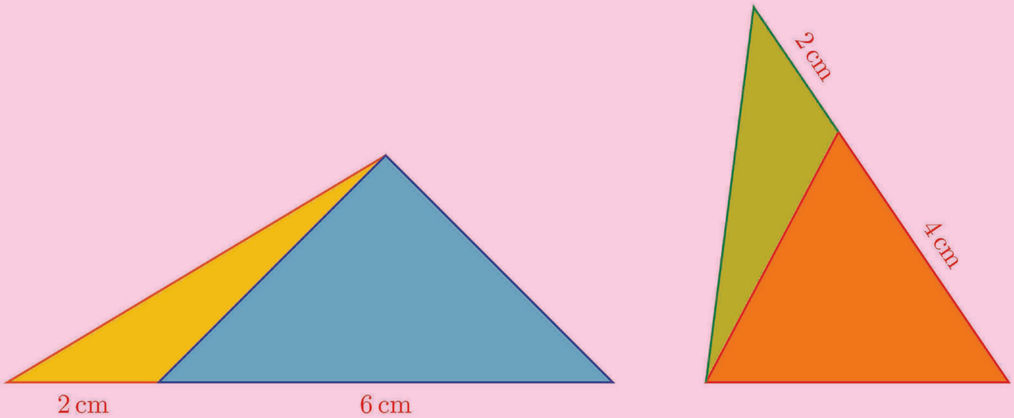
$$\text{Area of } \Delta ACD = \frac{1}{2} h \times CD$$

From this we have

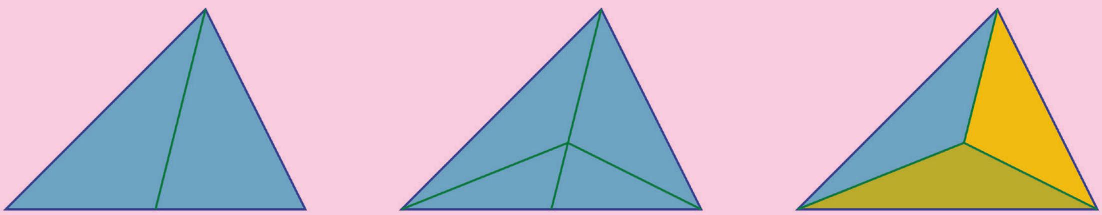
$$\frac{\text{Area of } \Delta ABD}{\text{Area of } \Delta ACD} = \frac{BD}{CD}$$

Thus the ratio of these areas is the same as the ratio of the lengths of BD and CD.
So how do we divide a triangle into two triangles of the equal areas?
What if we want the area of one part equal to twice the area of the other?

(1) In each of the two pictures below, a triangle is divided into two smaller triangles. Find what parts of the area of the whole triangle are the areas of the small triangles, in each picture:

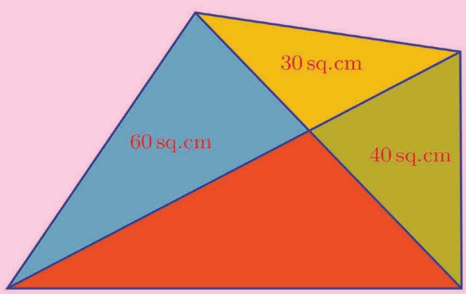


(2) The top vertex of a triangle is joined to the midpoint of the bottom side, and then the point dividing this line in the ratio 2 : 1 is joined to the other two vertices, as shown in the pictures below:



What fraction of the area of original large triangle are the areas of three small triangles in the third picture?

(3) The picture shows a quadrilateral divided into four triangles by its diagonals. The areas of three of these are marked in the picture:



Find the area of the whole quadrilateral.

9

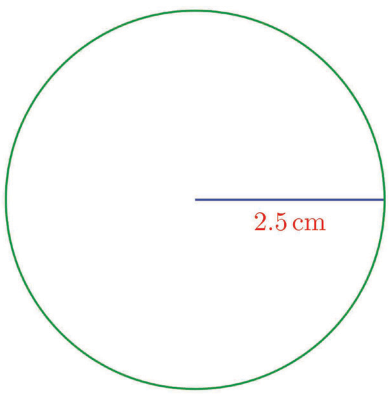
CIRCLES

Diameter

You have drawn many circles with various radii. How do you draw a circle of radius 2.5 centimetres?

First draw a 2.5 centimetres long line:

Now fix the point of the compass at one end of the line, stretch the other arm of the compass to the other end and draw the circle:



Can you draw a circle of radius 2.25 centimetres like this?

Recall how you drew an equilateral triangle of sides 3.25 centimetres in the chapter **Bisectors**. This was done by bisecting a 6.5 centimetres long line, right?

In the same way, we can get a 2.25 centimetres long line by bisecting a 4.5 centimetres long line, right?

Centre and radius

Do you remember what we said about circles in Class 5? (The section **Circle math** of the lesson **Lines and Circles**)

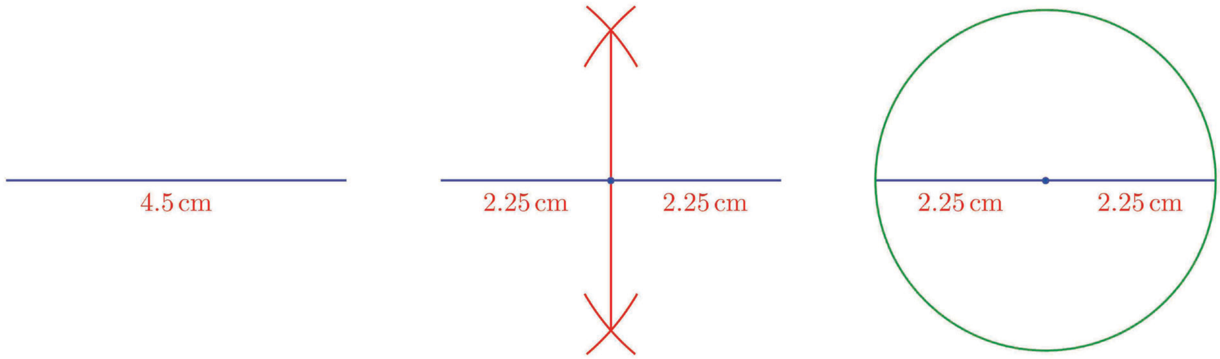
To draw a circle, we fix the point of the compass at one point and rotate the point of the pencil around it, right? Throughout this orbit, the point of the pencil is at the same distance from the compass point. We state this in mathematical language like this:

> A circle is the path traced by a moving point, keeping the same distance from a fixed point.

- The unmoving point in the middle is the centre of the circle.
- The unchanging distance between the unmoving point and the rotating point is the radius of the circle.

In GeoGebra, mark a point A, select Segment with Given Length and click on it; in the dialogue window, give 2.5 as Length to get the line AB of length 2.5. Right click on B and select Trace On and Animation On. What is the path that B traces?

And any line we can bisect by drawing the perpendicular bisector:



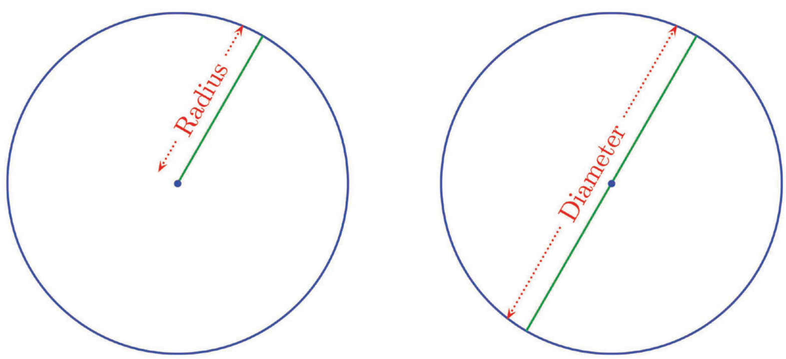
Here we drew the circle using a line twice as long as the radius. Double the radius of a circle is called the *diameter* of the circle.

For example, the radius of the circle we drew above is 2.25 centimetres and its diameter is 4.5 centimetres.

A line joining the centre of a circle to a point on the circle is also called *radius*, isn't it?

To draw a circle with a certain point as centre and a specified number as radius in GeoGebra, select Circle: Centre & Radius from the circle menu and click on the point. In the dialogue window, give the number as Radius. To draw a circle with a certain point as centre and passing through another point, select Circle with Centre through Point and click on the designated centre first and then on the other point.

Like this, a radius extended to the other side till it meets the circle again is also called a diameter.

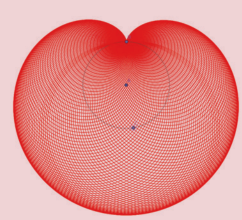


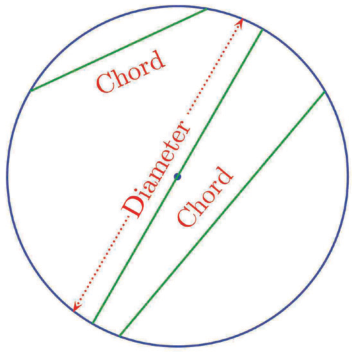
So diameter can also be described as a line joining two points on a circle and passing through the centre of the circle.

Any line joining two points on a circle is called a *chord*, in general.

Thus diameters are chords that pass through the centre of the circle.

Draw the circle centred at a point A and radius 3 in GeoGebra. Mark two points B and C on the circle and draw the circle centred at C and passing through B. Give a colour to this circle, by right clicking on it and selecting Object properties and choosing Color in the resulting window. Tick Trace On box for the circle and Animation On box for the point C.





Draw a circle centred at a point A and of radius 3 in GeoGebra and mark points B and C on it. Use Segment to join these points B and C to get a chord. Select Distance or Length and click on the chord to get its length. Move C around the circle. What is the maximum length of the chord? When does it become maximum?

We see another thing in the picture. The longest chord in the picture is a diameter.

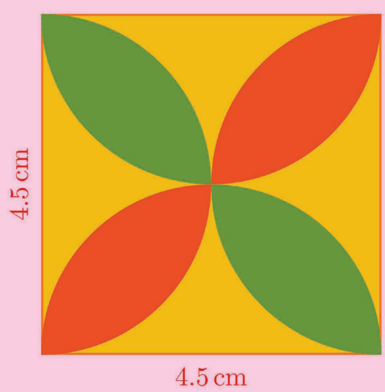
So in what all ways can we describe diameter of a circle?

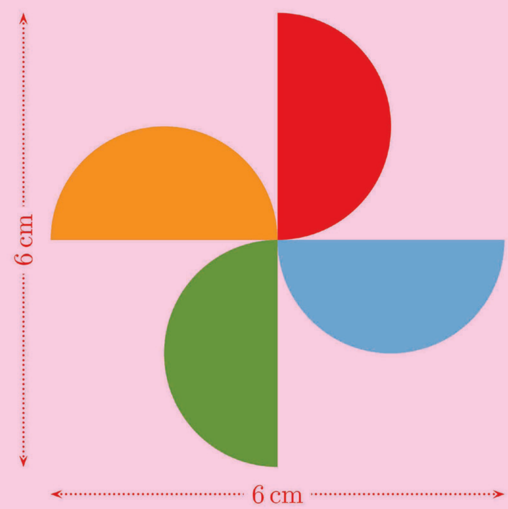
Diameter of a circle is

- **Double the radius**
- **Chord through the centre**
- **Chord of maximum length**

(1) Draw a circle of radius 2.75 centimetres.

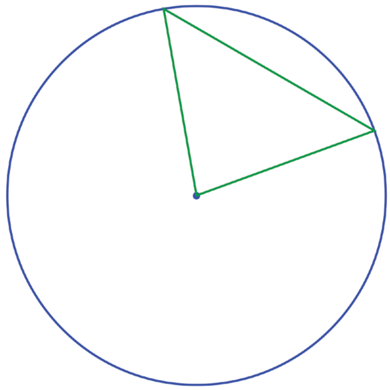
(2) Draw the pictures below:





Chords

If we join two ends of a chord to the centre of a circle, we get a triangle:

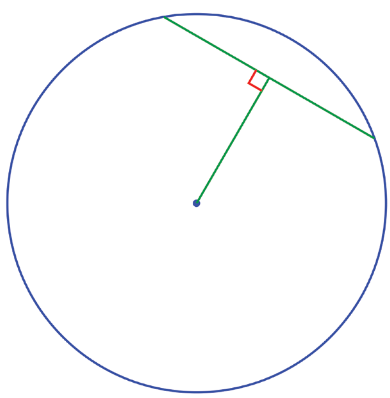


One side of this triangle is the chord itself; and the other two sides are radii of the circle and so equal.

Thus it is an isosceles triangle and we have seen in the lesson **Equal Triangles** that in any isosceles triangle, the perpendicular from the vertex joining the equal sides to the opposite side bisects this side (section **Isosceles Triangle**).

Here the vertex joining the equal sides is the centre of the circle, and the opposite side is the chord. So, the result about isosceles triangle translates to this result on circles:

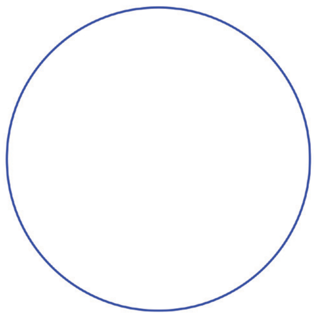
The perpendicular from the centre of a circle to a chord bisects the chord



As in the section **Isosceles triangle** of the lesson **Equal Triangles**, this also can be stated in two other ways:

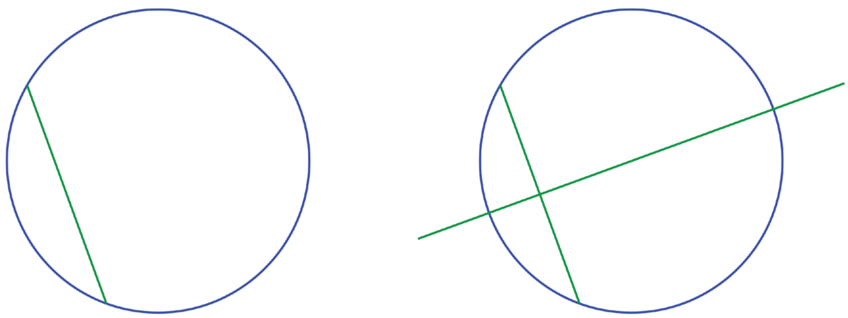
- **The line joining the midpoint of a chord and the centre of the circle is the perpendicular bisector of the chord**
- **The perpendicular bisector of a chord passes through the centre of the circle**

Using this, we can locate the centre of a circle. Draw a circle using a bangle or the lid of a small round tin:

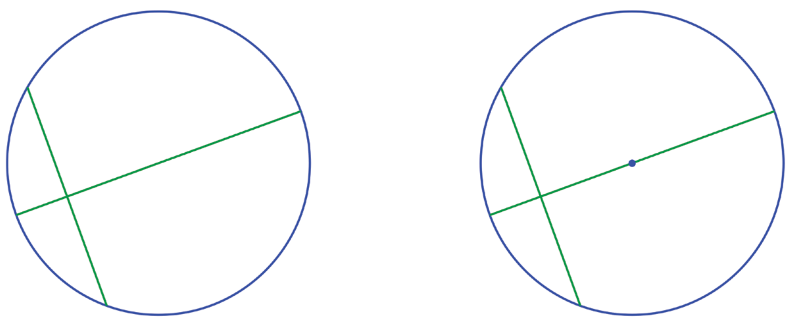


Draw a circle centred at a point A in GeoGebra and draw a chord BC in it. Select Perpendicular Bisector and click on the line to draw its perpendicular bisector. Does it pass through the centre? Change the positions of B and C and check.

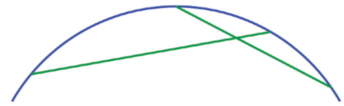
Its centre is on the perpendicular bisector of every chord. So, let's first draw a chord and its perpendicular bisector:



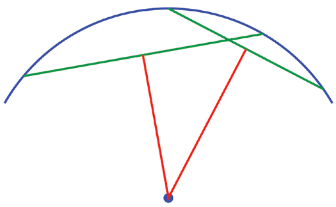
This perpendicular bisector passes through the centre of the circle. So if we erase the parts of it outside the circle, we get a chord through the centre; that is, a diameter. Now we can mark the centre as the midpoint of this diameter.

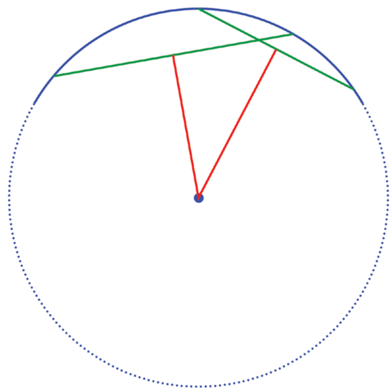


Now even if we get only a part of a circle such as a piece of a bangle, we can still locate the centre and thus draw the whole circle. For this, first draw two chords within this part:

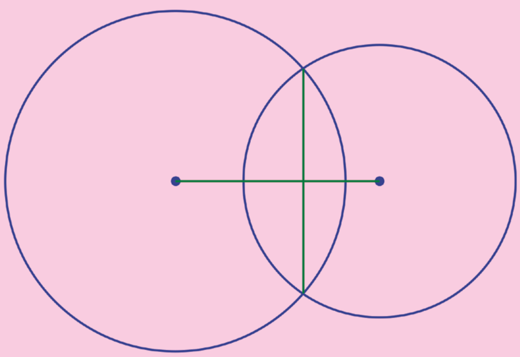


The centre of the circle must be on the perpendicular bisectors of both these chords; that is, the centre of the circle is the point where these bisectors intersect:





(1) In the picture, the horizontal line joins the centres of the circles and the vertical line joins the points of intersection of the circles.



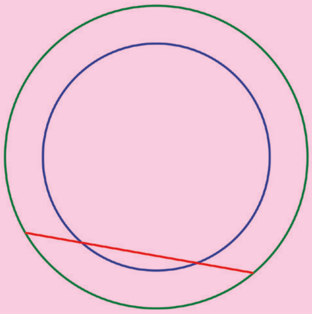
Write the reasons for the claims below:

(i) The perpendicular bisector of the vertical line passes through the centres of both the circles.

(ii) The horizontal line is the perpendicular bisector of the vertical line.

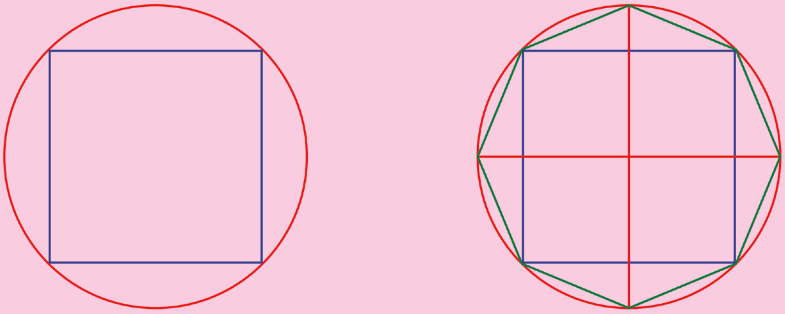
What general principle do we get from this?

(2) The picture shows two circles with the same centre and a line.



Prove that the portions of the line between the circles on both sides are equal.

(3) Draw a square and the circle through all four of its vertices. Join the vertices of the square and the points where the diameters parallel to the sides of the square meet the circle, to draw another polygon.

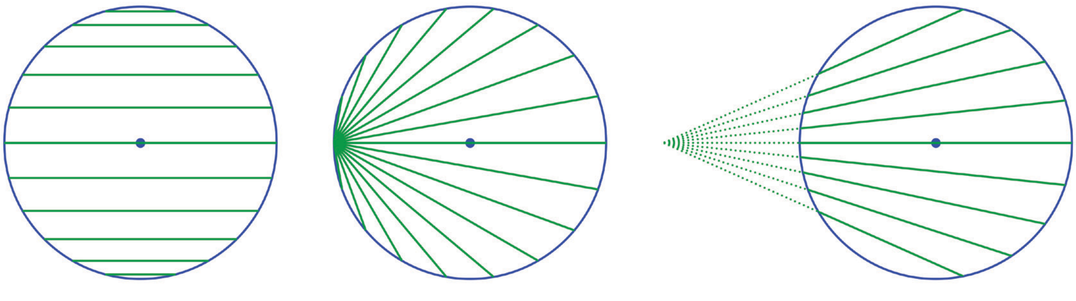


Prove that it is a regular octagon.

Equal chords

We have seen that diameters are chords through the centre of the circle; and also they are the chords of maximum length.

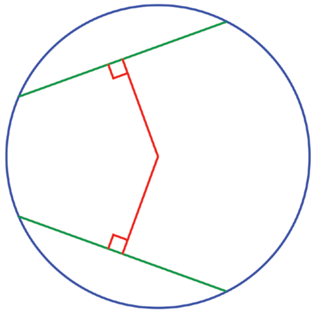
As chords move away from the centre, they become shorter:



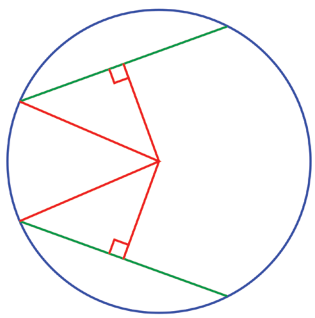
Whether the chords slide away from the centre or rotate away, doesn't it seem that chords at the same distance from the centre are of the same length?

How do we make sure of this?

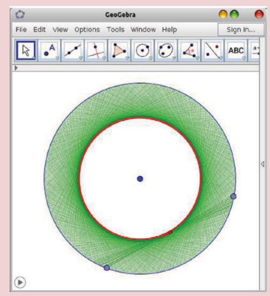
See this picture:



Two chords at the same perpendicular distance from the centre of the circle. To show that they are of the same length, join one end of each to the centre of the circle.



Draw a circle in GeoGebra and mark two points on it. Draw the chord joining these points and mark its midpoint. Tick Animation On for the endpoints and Trace On for the midpoint. What is the path traced by the midpoint? Why is this so? Tick Trace On for the chord also and see what happens. The picture can be made prettier by colouring the chord.



In the two right triangles we now get, the hypotenuses are equal, being radii of the circle. One pair of perpendicular sides are the equal perpendiculars to the chords. So by Pythagoras Theorem, the third sides are also equal.

These third sides are the pieces cut by perpendiculars from the centre. So they are half of the chords. Thus half the chords are equal and thus the chords themselves are equal.

Chords of a circle at equal distances from the centre have equal length

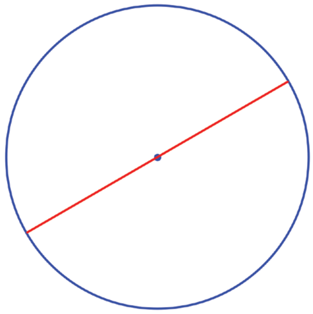
Does it hold the other way round?

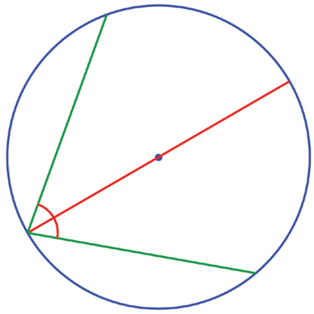
That is, are two equal chords of a circle are at equal distance from the centre?

Now look at the second of the three pictures seen at the beginning of this section. The length of chords drawn through the same point change, as the angles made with the diameter through the point change, right?

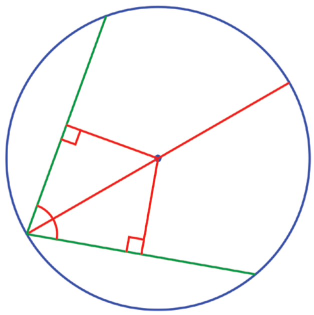
Are two such chords, making the same angle with the diameter on either side, are of the same length?

See these pictures. A diameter of a circle, and two chords making the same angle on either side of it are drawn:





To see whether they have the same length, we draw perpendiculars from the centre of the circle to these chords:



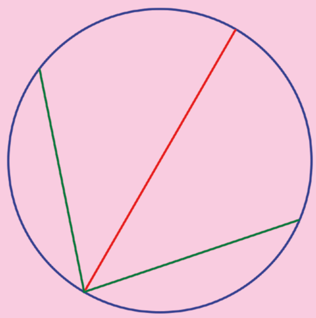
In the two right triangles we thus get, two of the angles are the same and so the third angles are also the same. And they have the same hypotenuse also. Since one side and the two angles on it are the same, the other two sides of the triangles are also equal (the section **Two angles** of the lesson **Equal Triangles**).

The green sides of the triangles are half the chords. Since they are equal, so are the chords themselves.

On the other hand, if we draw chords of the same length from one end of a diameter, would they make the same angle with the diameter?

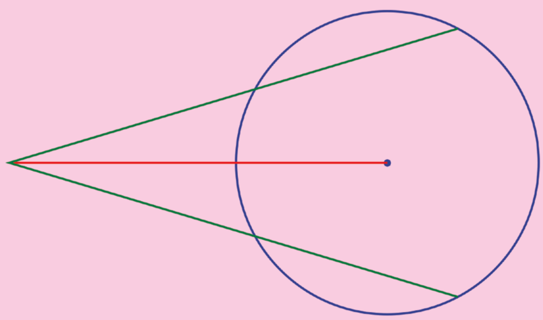
(1) Prove that chords of the same length in a circle are at the same distance from the centre of the circle.

(2) The picture shows two chords of the same length through one end of a diameter:



Prove that the angles which the chords make with the diameter are equal.

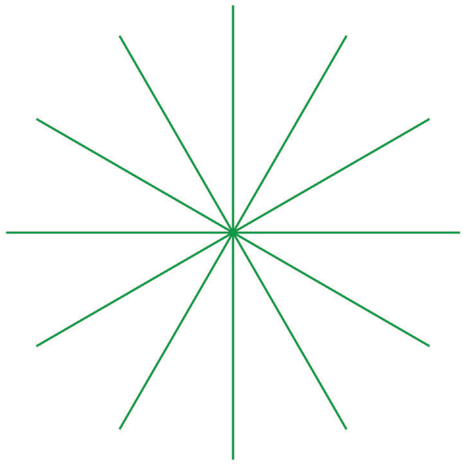
(3) In the picture, two chords of the same length in a circle are extended to meet at a point:



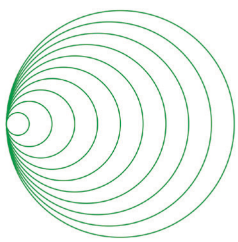
Prove that these lines make the same angle with the line joining the centre of the circle and the points where the lines meet.

Lines and circles

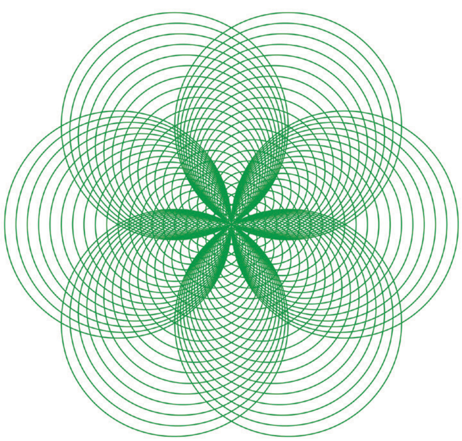
We can draw several lines passing through a single point:



We can also draw many circles passing through the same point: we can choose any point as the centre and the distance to this fixed point as the radius.



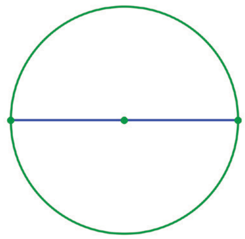
We can draw pretty patterns by drawing more circles:



What about two fixed points? They can be joined by a line. Can we draw a circle passing through these points?

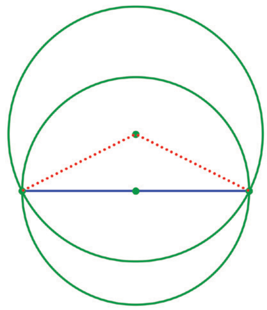
Mark two points in your notebook. How do you draw a circle passing through both?

An easy solution is to draw the circle with the line joining the two points as diameter:



Can you draw another?

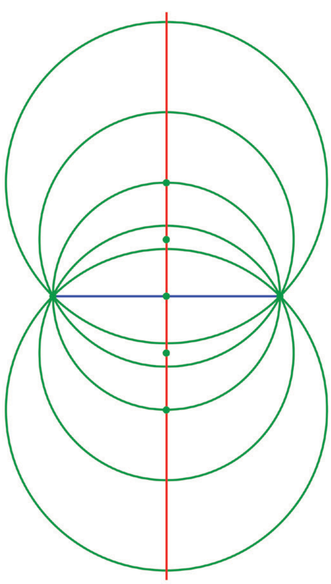
Can you choose the centre anywhere you like?



Since these points are to be on the circle, shouldn't they be at the same distance from the centre?

Thinking in reverse, this means the centre must be at the same distance from these points. The circle with such a point as centre and this distance as radius does pass through these points:

Since the centre is at the same distance from these points, it must be on the perpendicular bisector of the line joining these points; on the other hand, with any point on this bisector as centre, we can draw a circle passing through these points:

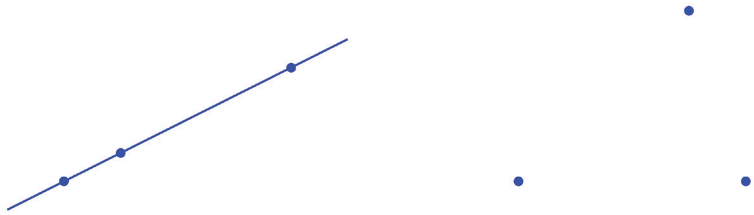


Draw a line AB and its perpendicular bisector in GeoGebra. Mark a point C on the bisector and draw a circle centred at C and passing through A. Set Trace On for the circle and Animation On for C. What do you see?

Thus we can draw one and only one line through two points, but several circles through two points.

What about three points?

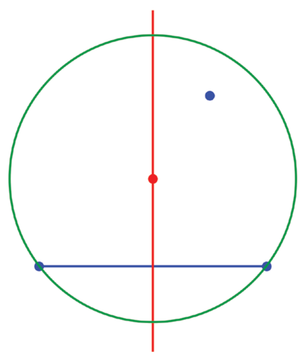
If the line through two of these passes through the other, then we have a line through all three points; otherwise it is not possible to draw a line through all three:



Can we draw a circle passing through any three points?

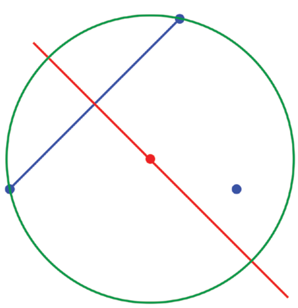
If the points are on the same line, we cannot. If they are not on the same line?

Before attempting this, let's think for a moment (Thought before deed is a mathematical habit).



Taking a point on the perpendicular bisector of the line joining any two of these points, we can draw the circle centred on it and passing through them.

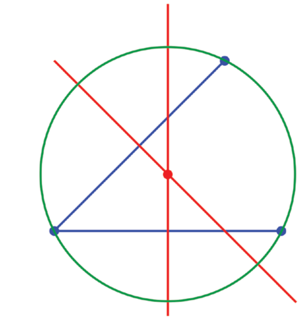
And if we take any point on the perpendicular bisector of the line joining another pair from these points, we can draw a circle through those two:



Mark three points in GeoGebra and draw the circle through them using Circle through 3 Points. Change the positions of these points. What happens when they are on the same line?

To get a circle through the first pair of points, the centre must be on the first perpendicular bisector and to get a circle through the second pair of points, the centre must be on the second perpendicular bisector.

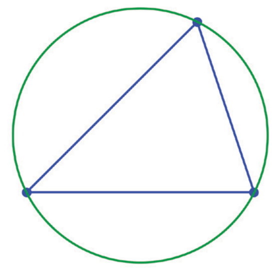
What if we take a point on both these bisectors? That is, their point of intersection?



We can draw the circle through all three points.

A circle can be drawn through any three points not on the same straight line

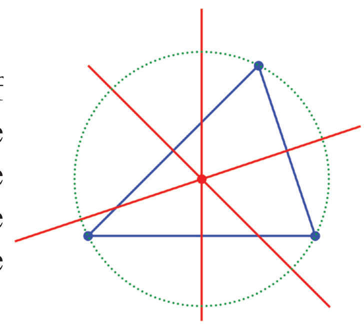
If we join the remaining pair of points, we get a triangle; and the circle is through all its vertices.



So, the above fact can be written like this also:

A circle can be drawn through all three vertices of a triangle

This circle is called the *circumcircle* of the triangle.



We can note another thing here. We found the centre of the circumcircle, called the *circumcentre*, by drawing the perpendicular bisectors of the bottom side and the left side of the triangle. Since the right side is also a chord of the circle, its perpendicular bisector also passes through the circumcentre.

In any triangle, the perpendicular bisectors of all three sides intersect at the same point

Draw triangles with sides as given below and draw the circumcircle of each.

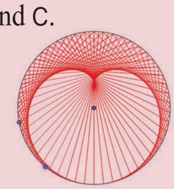
(i) Two sides of length 4 centimetres and 5 centimetres; the angle between them 60°.

(ii) Two sides of length 4 centimetres and 5 centimetres; the angle between them 120°.

(iii) Two sides of length 4 centimetres and 5 centimetres; the angle between them 90°.

(Note the change in the position of the circumcentre)

Draw a circle in GeoGebra and draw a chord BC in it. Right click on B and click on Object Properties. In the Algebra tab, give Speed as 3. Similarly set Speed as 6 for C. See Trace On for the chord and Animation On for B and C.



If we set Speed for B and C as 2 and 6, how is the picture changed? Experiment with other values for Speed.

10

Decimal Forms

Earlier forms

We have seen the decimal forms of some fractions in class 7 (The section **Decimal and fraction** of the lesson **Decimal Methods**). For example, we write,

$$\frac{3}{10} = 0.3$$

$$\frac{29}{100} = 0.29$$

$$\frac{347}{1000} = 0.347$$

In the other direction, we can write some numbers in decimal form as fractions with powers of 10 as denominators.

For example,

$$0.7 = \frac{7}{10}$$

$$0.91 = \frac{91}{100}$$

$$0.671 = \frac{671}{1000}$$

These can be split as the sum of fractions $\frac{1}{10}, \frac{1}{100}, \frac{1}{1000}, \ldots$ with powers of 10 as denominators:

$$0.91 = \frac{91}{100} = \frac{90}{100} + \frac{1}{100} = \frac{9}{10} + \frac{1}{100}$$

$$0.671 = \frac{671}{1000} = \frac{600}{1000} + \frac{70}{1000} + \frac{1}{1000} = \frac{6}{10} + \frac{7}{100} + \frac{1}{1000}$$

So, what is the meaning of 0.03?

$$0.03 = \frac{0}{10} + \frac{3}{100} = \frac{3}{100}$$

What about 0.0203?

$$0.0203 = \frac{0}{10} + \frac{2}{100} + \frac{0}{1000} + \frac{3}{10000} = \frac{200}{10000} + \frac{3}{10000} = \frac{203}{10000}$$

Write in fractional form, the numbers given in decimal forms below:

(i) 0.1 (ii) 0.01 (iii) 0.11 (iv) 0.101 (v) 0.0101

Some other fractions

Some fractions with denominators not a power of 10 can be written in such a form.

For example, since 10 = 2 × 5, we have

$$\frac{1}{2} = \frac{5}{10} = 0.5$$

$$\frac{1}{5} = \frac{2}{10} = 0.2$$

From these we can write

$$\frac{2}{5} = \frac{2 \times 2}{5 \times 2} = \frac{4}{10} = 0.4$$

$$\frac{3}{5} = \frac{3 \times 2}{5 \times 2} = \frac{6}{10} = 0.6$$

$$\frac{4}{5} = \frac{4 \times 2}{5 \times 2} = \frac{8}{10} = 0.8$$

and so on. We could do this because 2 and 5 are factors of 10. But then, how do we write $\frac{1}{4}$ in decimal form?

4 is not a factor of 10, but it is a factor of 100, isn't it?

$$4 \times 25 = 100$$

Using this, we can write

$$\frac{1}{4} = \frac{1 \times 25}{4 \times 25} = \frac{25}{100} = 0.25$$

$$\frac{3}{4} = \frac{3 \times 25}{4 \times 25} = \frac{75}{100} = 0.75$$

and so on.

Also we can compute

$$\frac{1}{25} = \frac{1 \times 4}{25 \times 4} = \frac{4}{100} = 0.04$$

$$\frac{2}{25} = \frac{2 \times 4}{25 \times 4} = \frac{8}{100} = 0.08$$

$$\frac{13}{25} = \frac{13 \times 4}{25 \times 4} = \frac{52}{100} = 0.52$$

What about $\frac{1}{8}$?

8 is a factor of neither 10 nor 100.

But since 8 = 2 × 2 × 2, we can multiply it by 5 three times to get the product of three 10's, right?

In mathematical terms,

$$2^3 \times 5^3 = (2 \times 5)^3 = 10^3 = 1000$$

That is,

$$8 \times 125 = 1000$$

Using this, we get

$$\frac{1}{8} = \frac{1 \times 125}{8 \times 125} = \frac{125}{1000} = 0.125$$

$$\frac{3}{8} = \frac{3 \times 125}{8 \times 125} = \frac{375}{1000} = 0.375$$

and so on; also we get

$$\frac{1}{125} = \frac{1 \times 8}{125 \times 8} = \frac{8}{1000} = 0.008$$

$$\frac{3}{125} = \frac{3 \times 8}{125 \times 8} = \frac{24}{1000} = 0.024$$

$$\frac{13}{125} = \frac{13 \times 8}{125 \times 8} = \frac{104}{1000} = 0.104$$

and so on.

Now what can we do about $\frac{3}{160}$?

First we factorize the denominator:

$$160 = 32 \times 5 = 2^5 \times 5$$

What power of 10 can we get as a multiple of this?

For by what number should we multiply it ?

$$160 \times 5^4 = (2^5 \times 5) \times 5^4 = 2^5 \times 5^5 = (2 \times 5)^5 = 10^5$$

Using this, we get

$$\frac{3}{160} = \frac{3 \times 5^4}{160 \times 5^4} = \frac{3 \times 625}{100000} = \frac{1875}{100000} = 0.01875$$

So, what kind of fractions can we write in the decimal form like this?

(1) Write the fractions below in the decimal form:

(i) $\frac{3}{20}$ (ii) $\frac{3}{40}$ (iii) $\frac{13}{40}$ (iv) $\frac{7}{80}$ (v) $\frac{5}{16}$

(2) Find the decimal form of the sums below:

(i) $\frac{1}{5} + \frac{1}{25} + \frac{1}{125}$

(ii) $\frac{1}{5} + \frac{1}{5^2} + \frac{1}{5^3} + \frac{1}{5^4}$

(iii) $\frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3}$

(3) A two-digit number divided by another two-digit number gives 5.875. What are the numbers?

New forms

We saw that for some fractions with denominators not a power of 10, we can find such a form and then they can be written in the decimal form.

Can we do this for $\frac{1}{3}$?

$\frac{1}{3}$ has various forms:

$$\frac{1}{3} = \frac{2}{6} = \frac{3}{9} = \frac{4}{12} = \dots$$

and so on.

In all these, the denominator is a multiple of 3.

So in none of these forms is the denominator a power of 10, since all powers of 10 leave remainder 1 on division by 3.

$$10 = (3 \times 3) + 1$$

$$100 = (3 \times 33) + 1$$

$$1000 = (3 \times 333) + 1$$

So, $\frac{1}{3}$ cannot be written in a decimal form seen so far.

But then, we can write some fractions close to $\frac{1}{3}$ in decimal form.

For that, we write in fractional form, the powers of 10 written as multiples of 3 and remainder, as above:

$$\frac{10}{3} = 3 + \frac{1}{3}$$

$$\frac{100}{3} = 33 + \frac{1}{3}$$

$$\frac{1000}{3} = 333 + \frac{1}{3}$$

We can write them in a slightly different way like this:

$$\frac{1}{3} \times 10 = 3 + \frac{1}{3}$$

$$\frac{1}{3} \times 100 = 33 + \frac{1}{3}$$

$$\frac{1}{3} \times 1000 = 333 + \frac{1}{3}$$

Look at the first one. This means $3 + \frac{1}{3}$ is 10 times $\frac{1}{3}$. So, $\frac{1}{3}$ is $\frac{1}{10}$ times of $3 + \frac{1}{3}$ (The section Times and parts of the lesson Reciprocals in the Class 7). That is,

$$\frac{1}{3} = \frac{1}{10} \times \left(3 + \frac{1}{3}\right)$$

We have seen in class 7 that to multiply a sum, we have to multiply each of the numbers and add (The section **Addition and subtraction and then multiplication** of the lesson **Shorthand Math**). According to this, how do we write $\frac{1}{10} \times \left(3 + \frac{1}{3}\right)$?

Thus,

$$\frac{1}{3} = \frac{3}{10} + \frac{1}{30}$$

Similarly, how can we write $\frac{1}{3}$, using $\frac{100}{3}$ and $\frac{1000}{3}$ written as above?

$$\frac{1}{3} = \frac{33}{100} + \frac{1}{300}$$

$$\frac{1}{3} = \frac{333}{1000} + \frac{1}{3000}$$

Let's write all these as the difference of $\frac{1}{3}$ with $\frac{3}{10}, \frac{33}{100}, \frac{333}{1000}$

$$\frac{1}{3} - \frac{3}{10} = \frac{1}{30}$$

$$\frac{1}{3} - \frac{33}{100} = \frac{1}{300}$$

$$\frac{1}{3} - \frac{333}{1000} = \frac{1}{3000}$$

$\frac{1}{300}$ is less than $\frac{1}{30}$ and $\frac{1}{3000}$ is still less.

Thus the difference between $\frac{1}{3}$ and the fractions $\frac{3}{10}, \frac{33}{100}, \frac{333}{1000}$ becomes smaller and smaller.

We can continue further:

$$\frac{1}{3} - \frac{3333}{10000} = \frac{1}{30000}$$

Summarizing we have found this

The numbers $\frac{3}{10}, \frac{33}{100}, \frac{333}{1000}, \ldots$ get closer and closer to the number $\frac{1}{3}$

Using the decimal forms of $\frac{3}{10}, \frac{33}{100}, \frac{333}{1000}, \ldots$ we can write this in the form:

The numbers 0.3, 0.33, 0.333, . . . get closer and closer to the number $\frac{1}{3}$.

We write this in a short form as

$$\frac{1}{3} = 0.333\ldots$$

It must be especially noted that the decimal 0.333 . . . used here is different from the decimal forms seen so far.

The numbers written in decimal form earlier were those fractions with powers of 10 as the denominators. For example 0.3 is the decimal form of $\frac{3}{10}$ and 0.33 is the decimal form of $\frac{33}{100}$.

But the decimal form 0.333 . . . does not indicate any fraction with denominator a power of 10. But the fraction which is approached closer and closer by a succession of fractions with powers of 10 as denominators. This fraction is $\frac{1}{3}$, as seen above, and so we call it the decimal form of $\frac{1}{3}$.

Let's look at another example. None of the many forms of the fraction $\frac{1}{6}$ has a power of 10 as its denominator (Why?)

We can find the new kind of decimal form of this also. For this, we first divide 10, 100, 1000, . . . by 6 as before and write:

$$\frac{10}{6} = \frac{5}{3} = 1 + \frac{2}{3}$$

$$\frac{100}{6} = \frac{50}{3} = 16 + \frac{2}{3}$$

$$\frac{1000}{6} = \frac{500}{3} = 166 + \frac{2}{3}$$

Using this, we can write $\frac{1}{6}$ as the sum of a fraction with a power of 10 as denominator and another small fraction in various ways:

$$\frac{1}{6} = \frac{1}{10} + \frac{2}{30} = \frac{1}{10} + \frac{1}{15}$$

$$\frac{1}{6} = \frac{16}{100} + \frac{2}{300} = \frac{16}{100} + \frac{1}{150}$$

$$\frac{1}{6} = \frac{166}{1000} + \frac{2}{3000} = \frac{166}{1000} + \frac{1}{1500}$$

From this we get fractions with powers of 10 as denominator, which get closer and closer to $\frac{1}{6}$:

$$\frac{1}{6} - \frac{1}{10} = \frac{1}{15}$$

$$\frac{1}{6} - \frac{16}{100} = \frac{1}{150}$$

$$\frac{1}{6} - \frac{166}{1000} = \frac{1}{1500}$$

That is,

The numbers $\frac{1}{10}, \frac{16}{100}, \frac{166}{1000}, \ldots$ (with decimal forms 0.1, 0.16, 0.166, ...) get closer and closer to the number $\frac{1}{6}$.

Again, we shorten this as a decimal form:

$$\frac{1}{6} = 0.1666\ldots$$

In dividing 10, 100, 1000, . . . to find a fractional form of a number, we need not do each division from the beginning; we can do one division as a continuation of the previous division.

For example, let's find the decimal form of $\frac{1}{7}$. First we divide 10 by 7 and write like this:

$$\frac{10}{7} = 1 + \frac{3}{7}$$

Now to compute $\frac{100}{7}$, we need only multiply this by 10 and write:

$$\frac{100}{7} = 10 + \frac{30}{7} = 10 + 4 + \frac{2}{7} = 14 + \frac{2}{7}$$

We can continue like this:

$$\frac{1000}{7} = 140 + \frac{20}{7} = 142 + \frac{6}{7}$$

$$\frac{10000}{7} = 1420 + \frac{60}{7} = 1428 + \frac{4}{7}$$

It is convenient to use powers of 10, so that we don't lose count of the zeros:

$$\frac{10}{7} = 1 + \frac{3}{7}$$

$$\frac{10^2}{7} = 14 + \frac{2}{7}$$

$$\frac{10^3}{7} = 142 + \frac{6}{7}$$

$$\frac{10^4}{7} = 1420 + \frac{60}{7} = 1428 + \frac{4}{7}$$

$$\frac{10^5}{7} = 14280 + \frac{40}{7} = 14285 + \frac{5}{7}$$

$$\frac{10^6}{7} = 142850 + \frac{50}{7} = 142857 + \frac{1}{7}$$

Need we continue? Let's take a moment to think. The next division is

$$\frac{10^7}{7} = 1428570 + \frac{10}{7}$$

In this, $\frac{10}{7}$ is something we have already computed. So,

$$\frac{10^7}{7} = 1428571 + \frac{3}{7}$$

What if we continue further?

The same operations on fractions done earlier repeat as $\frac{30}{7} = 4 + \frac{2}{7}$ then $\frac{20}{7} = 2 + \frac{6}{7}$ and so on and in the whole number part, the same digits got earlier repeats as 1428571, 14285714, 142857142 and so on.

Thus the decimal form of $\frac{1}{7}$ is the repetition of the six-digit block 142857. In other words

$$\frac{1}{7} = 0.142857142857...$$

See the computation of $\frac{2}{7}$ like this:

$$\frac{20}{7} = 2 + \frac{6}{7}$$

$$\frac{200}{7} = 20 + \frac{60}{7} = 28 + \frac{4}{7}$$

$$\frac{2000}{7} = 280 + \frac{40}{7} = 285 + \frac{5}{7}$$

$$\frac{20000}{7} = 2850 + \frac{50}{7} = 2857 + \frac{1}{7}$$

The next division is $\frac{10}{7}$, right?

And we have computed this earlier.

Also, the next two divisions are $\frac{30}{7}$ and $\frac{20}{7}$ as in the case of $\frac{1}{7}$.

Then we get $\frac{60}{7}$ we did earlier.

So, the decimal form of $\frac{2}{7}$ is the repetition of which six-digit block?

$$\frac{2}{7} = 0.285714285714...$$

Now try to compute the decimal forms of $\frac{3}{7}$ and $\frac{4}{7}$.

(1) Find fractions with powers of 10 as denominator which get closer and closer to each of the fractions below and write the decimal form of each:

(i) $\frac{5}{6}$ (ii) $\frac{1}{9}$ (iii) $\frac{1}{11}$

(2) (i) Prove that the numbers $\frac{1}{10}, \frac{11}{100}, \frac{111}{1000}, ...$ get closer and closer to $\frac{1}{9}$.

(ii) Find the decimal forms of $\frac{1}{9}, \frac{2}{9}, \frac{3}{9}, \frac{4}{9}, \frac{5}{9}, \frac{6}{9}, \frac{7}{9}, \frac{8}{9}$

(iii) What is the decimal form of $\frac{2}{3}$?

(3) Compute the following and write answers in decimal form:

(i) $0.111... + 0.222...$ (ii) $0.333... + 0.777...$ (iii) $0.333... \times 0.666...$

(4) For each of the fractions below, write the decimal form of a fraction with difference less than $\frac{1}{1000}$.

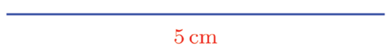
(i) $\frac{1}{3}$ (ii) $\frac{1}{6}$ (iii) $\frac{2}{3}$ (iv) $\frac{5}{6}$ (v) $\frac{1}{7}$

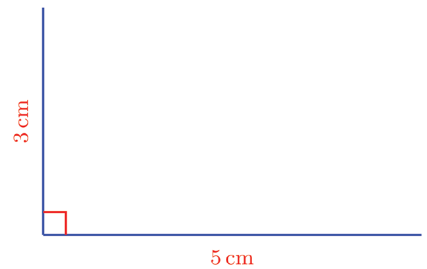
11

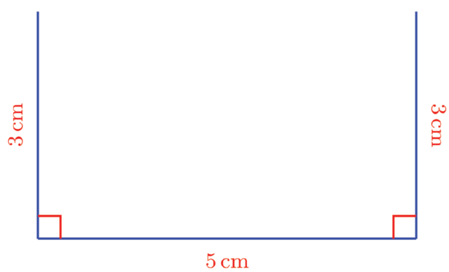
PARALLELOGRAMS

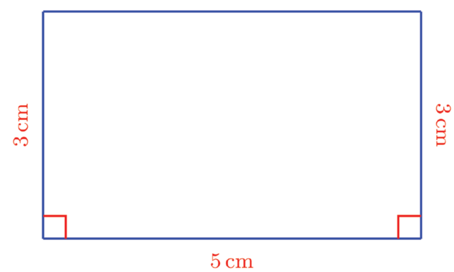
Sides and angles

To draw a rectangle, only the lengths of two sides need to be specified. For example, how do we draw a rectangle of sides 5 centimetres and 3 centimetres?









If we draw at the ends of a 5 centimetres line, perpendiculars of height 3 centimetres and join their upper ends, as done above, do we get a rectangle?

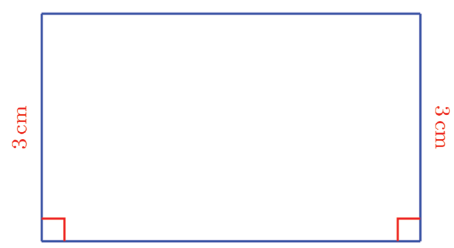
Will the length of the top side be also 5 centimetres?

We can clear this doubt like this:

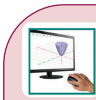
(i) Since the left and right sides make the same angle with the bottom line, they are parallel (The lesson **Parallel Line** in the Class 7 textbook).

(ii) Since the left and right sides are parallel and of the same length, the quadrilateral drawn is a parallelogram (The section **Two sides** of the lesson **Equal Triangles**).

(iii) Since the quadrilateral is a parallelogram, the top and bottom sides are of the same length (The section **Two angles** in the lesson **Equal Triangles**).

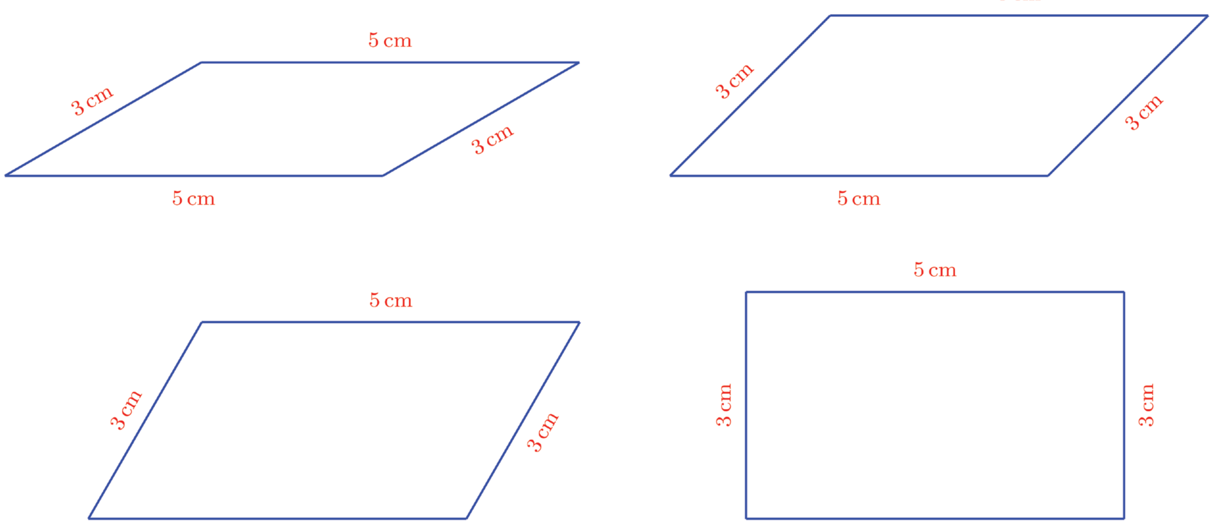


Now what about a parallelogram with two sides 5 centimetres and 3 centimetres? We can draw several, right?



Unchanging sides

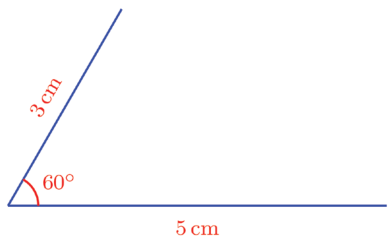
Draw a 5 centimetres long line in GeoGebra using Segment with Given Length tool. Using the same tool draw a line BC of length 3 centimetres. Change the position of C so that A, B, C are not on the same straight line. Draw the circle of radius 3 centimetres, centred at A. Draw the line parallel to BC through A and mark the point D where this line meets the circle. Join CD and hide the lines and the circle. Draw the parallelogram ABCD using the Polygon tool. By changing the position of C, can't we get different parallelograms with sides 5 centimetres and 3 centimetres long?



What makes them different from one another?

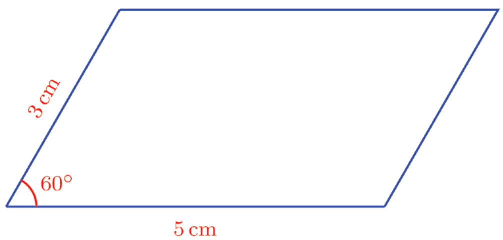
So, if in addition to specifying the lengths of two sides as 5 centimetres and 3 centimetres, what if we also say that the angle between the sides is 60° ?

First we draw a 5 centimetres line horizontally and draw a 3 centimetres long line at one of its ends, inclined at an angle of 60°.



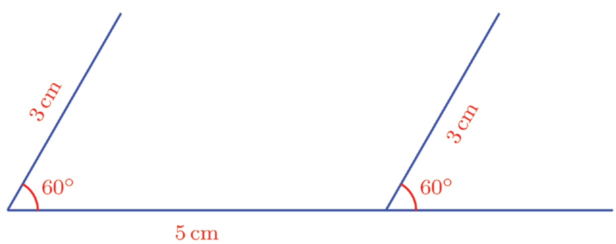
What next?

We draw the line parallel to the bottom line through the top end of the slanted line; and also the line parallel to the slanted line through the other end of the horizontal line and cut them off at the point where they meet (We have seen how parallel lines can be drawn using a scale and set square in the section **Non intersecting lines** of the lesson **Lines and Angles** in the Class 6 textbook).

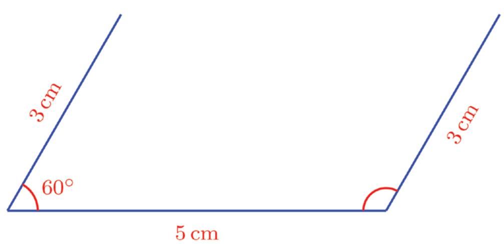


Is there an easier way to draw this? (This is indeed the easiest way, if we are doing it in GeoGebra).

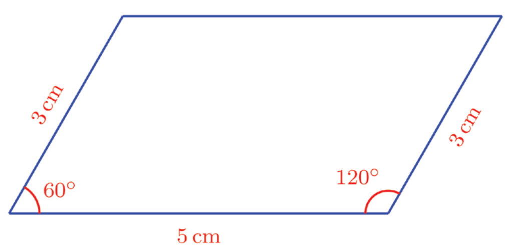
Isn't it enough if we extend the bottom line a bit to the right and draw a 3 centimetres long line from the right end, inclined at 60°?



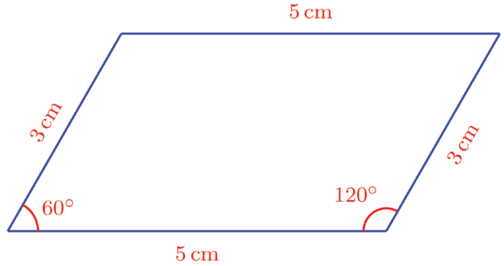
Instead of extending the bottom line, we can draw the right side by measuring off the angle on the other side:



What should be the measure of that angle?

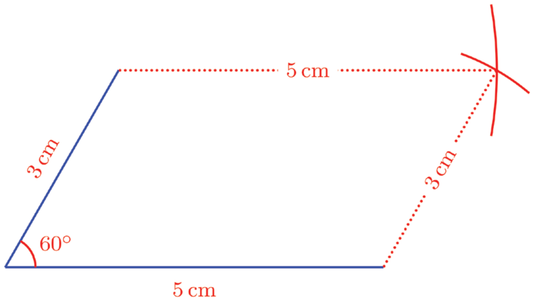


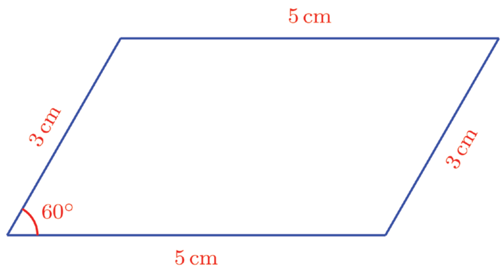
We can prove that the top side is also 5 centimetres long using the arguments used in the case of a rectangle.



After drawing a 5 centimetres long horizontal line and a line at an inclination of 60° through one of its ends, we can proceed in a different way to draw the parallelogram.

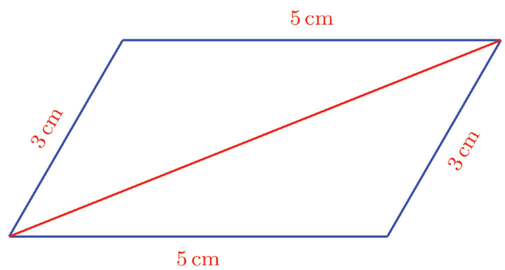
The fourth vertex of the parallelogram is 5 centimetres away from the top end of the slanted line, and 3 centimetres away from the right end of the horizontal line, isn't it?



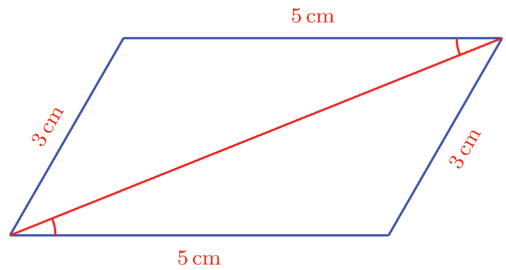


But then there is a question: are the pairs of opposite sides parallel?

To settle this, let's draw a diagonal of the quadrilateral drawn like this, to split it into two triangles:



In these two triangles, the lengths of two pairs of lines are the same; and the third side is the same line for both. So, the angles opposite to equal sides are also equal. That is, the angles marked in the picture below are equal:



These are the small angles made by the diagonal meeting the top and bottom sides.

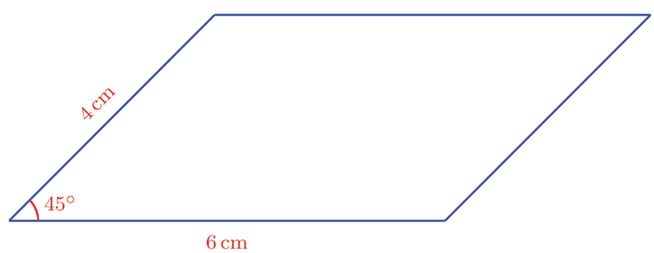
Since they are equal, these sides are parallel. Since they have the same length also, the quadrilateral is a parallelogram.

These arguments can be used to prove that a quadrilateral drawn like this with any lengths for the sides is a parallelogram.

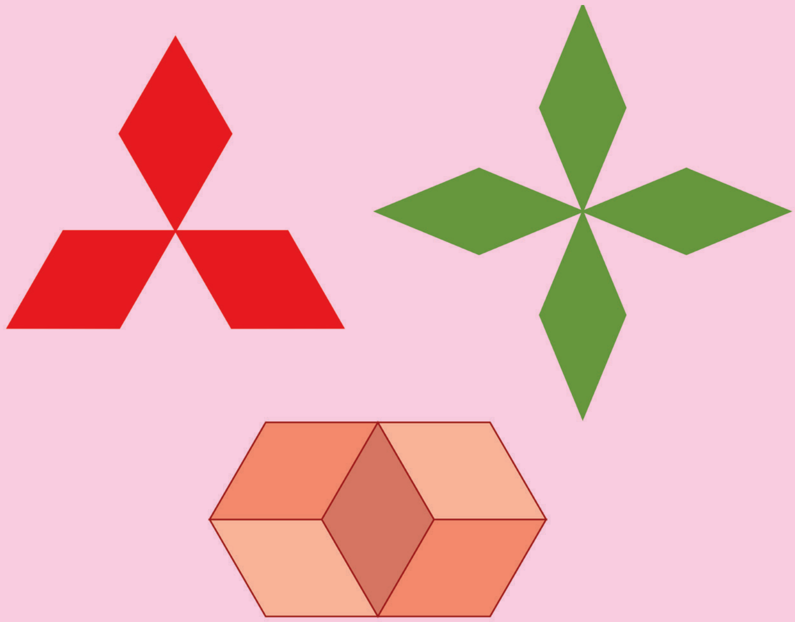
Thus we can reverse the assertion that pairs of opposite sides of a parallelogram are equal:

Any quadrilateral with pairs of opposite sides equal, is a parallelogram.

Thus there are many ways to draw a parallelogram with specified lengths of sides and specified angle between them. Use any of them to draw the parallelogram shown below:

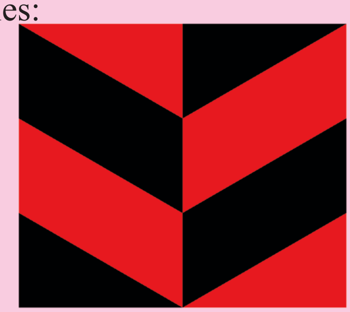


(1) The figures below are combinations of three, four and five equal rhombuses. Draw each and colour it.



(2) This picture is made up of three equal parallelograms and an equilateral triangle. Draw and colour it.

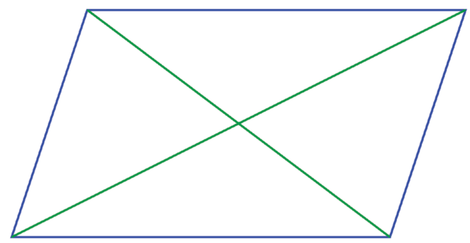
(3) Draw and colour this picture made up of four equal parallelograms and four equal right triangles:



Draw and colour these pictures in GeoGebra. You can use the grid to make drawing easier.

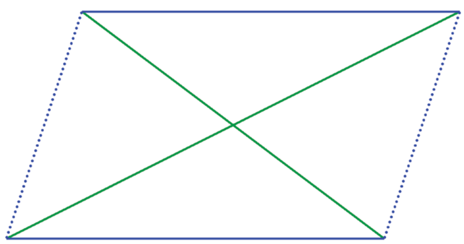
Diagonals

We have seen that the diagonals of any rhombus are perpendicular bisectors of each other and on the other hand, any quadrilateral in which diagonals are perpendicular bisectors of each other is a rhombus (the section **Rhombuses** of the lesson **Bisectors**).



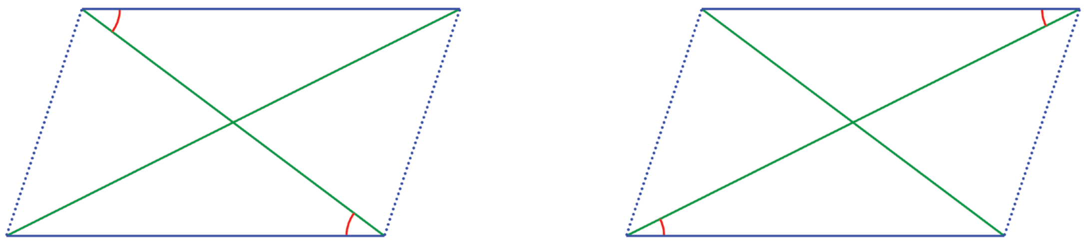
In a parallelogram with unequal sides, the diagonals are not mutually perpendicular:

But it is not difficult to see that they bisect each other. For this, note that the diagonals divide the parallelogram into four triangles and look at just the top and bottom triangles:



The blue sides of both triangles are equal, being the opposite sides of the parallelogram.

Again, since these sides are parallel, the small angles which they make with each diagonal are also equal. That is, the angles marked in each picture below are equal:

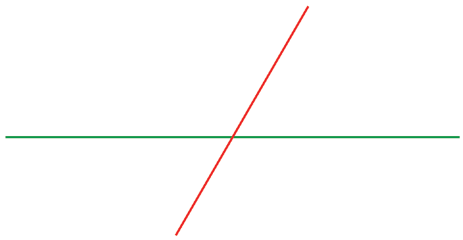


So, in these triangles one side and the angles on them are the same and so the sides opposite equal angles are equal.

Thus we can see from the above pictures that the parts into which each diagonal divides the other are equal. In other words, they bisect each other.

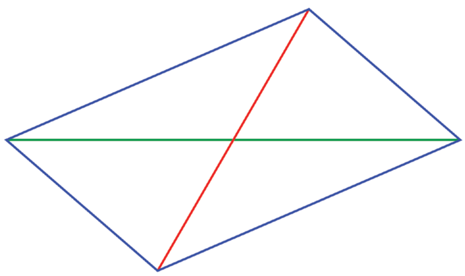
In the other direction, is any quadrilateral in which diagonals bisect each other, a parallelogram?

To check this, let's draw such a quadrilateral. First we draw a line and another line through its midpoint:

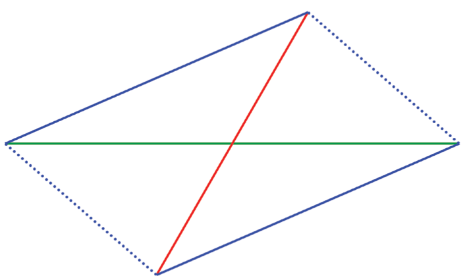


Draw a line AB in GeoGebra using the Segment tool and mark its midpoint C using the Midpoint or Centre tool. Draw a circle centred on C and mark a point D on it. Draw the line through C and D and mark the point E where it intersects the circle again. Hide the line and the circle and draw DE. Draw the quadrilateral ADBE. Is it a parallelogram? Mark the lengths of the sides and check. What happens to the quadrilateral as the diagonals AB and DE are equal? And when they are perpendicular to each other?

Next mark the same length on either side of the slanted line and draw a quadrilateral:



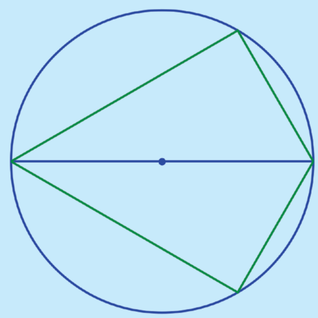
The quadrilateral is split into four triangles. Look at only the top left and bottom right triangles:



Because of the way we have drawn the picture, the green sides of the triangles are equal; and so are the red sides. And the angles between these sides are also equal, being opposite angles made by the intersecting diagonals (The section **Intersecting lines** of the lesson **Lines and Angles** in the Class 6 textbook).

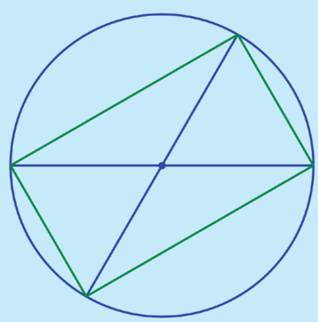
Rectangle in a circle

Draw a circle and a diameter. Mark a point on each half of the circle and join them to the ends of the diameter:



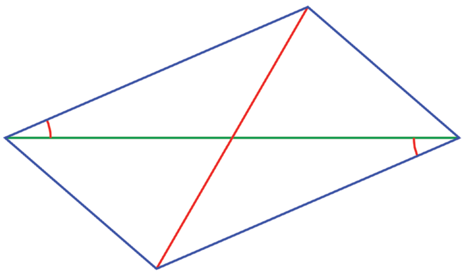
Such a quadrilateral may not be a rectangle. But the angles on either side of the diameter are right angles (Why?). What about the other two angles of the quadrilateral?

What if the right angle corners of the picture are at the ends of another diameter?



All the four angles are right angles and so the quadrilateral is a rectangle.

So, the third (blue) sides of the triangles are equal; and so are the other two angles opposite equal sides.



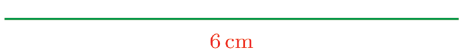
Thus in our quadrilateral, the top and bottom sides are equal; and the angles which they make with the green line are also equal:

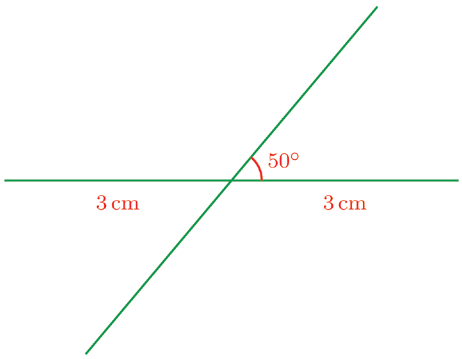
So, the blue lines are also parallel. Since two sides are equal and parallel, the quadrilateral is a parallelogram.

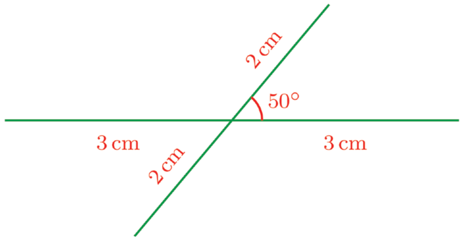
In any parallelogram, the diagonals bisect each other; on the other hand, any quadrilateral in which the diagonals bisect each other is a parallelogram

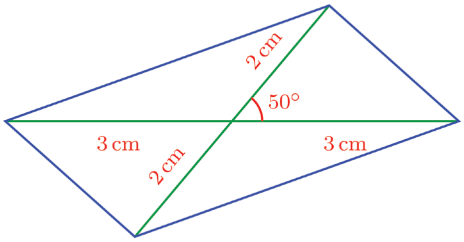
In a rhombus, the angle between the diagonals is a right angle. In parallelograms with unequal sides, this angle can be of different measures.

So, if the lengths of diagonals of a parallelogram and the angle between them are specified, we can draw a parallelogram according to these. For example, the parallelogram with lengths of diagonals 6 centimetres and 4 centimetres and the angle between them 50° can be drawn like this:



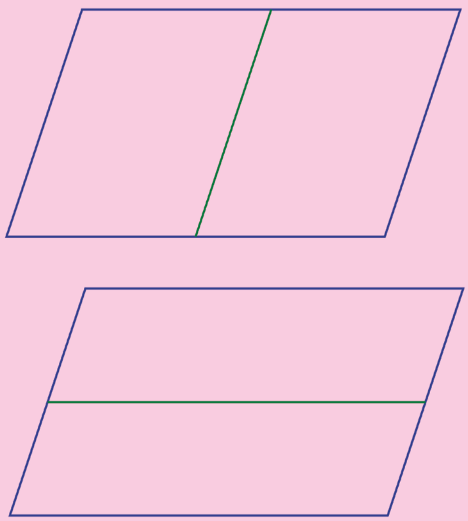




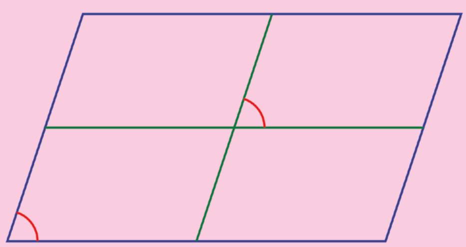


(1) Draw a parallelogram with lengths of diagonals 8 centimetres and 6 centimetres and the angle between them 60°.

(2) Prove that in a parallelogram, the line joining the midpoints of two opposite sides is parallel to the other two sides:



(3) The picture shows a parallelogram and the lines joining the midpoint of opposite sides:

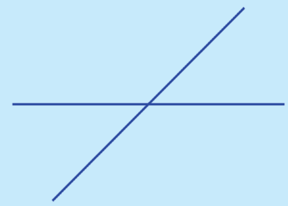


(i) Prove that these lines bisect each other.

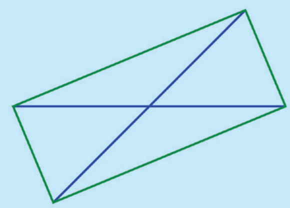
(ii) Prove that an angle between these lines is equal to an angle between the sides of the parallelogram.

Diagonals

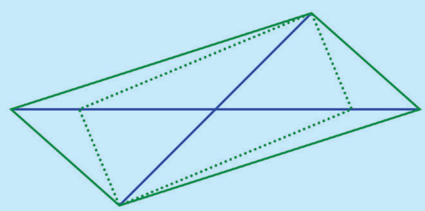
Draw two lines of the same length bisecting each other, but not perpendicular:



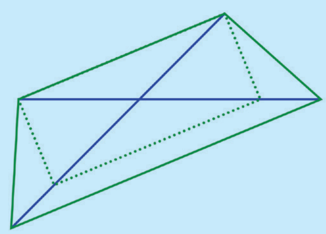
What type of quadrilateral do we get by joining the end points of these lines?



What happens if we extend one of the lines equally on either side and join the new end points?



Now instead of extending one of the lines of the first picture equally to either side, if we equally extend the horizontal line to the right and the slanted line downward, and join the end points, what figure do we get?



(4) Draw a parallelogram with lengths of diagonals 6.5 centimetres and 4.5 centimetres and the angle between them 70°.

(5) Prove that if the diagonals of a parallelogram are of the same length, then it is a rectangle.

(6) Draw a rectangle with length of diagonals 6 centimetres and the angle between them 60°.

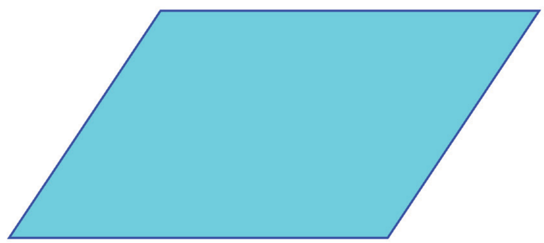
Area

The area of a rectangle is the product of the lengths of its sides. What about a parallelogram?

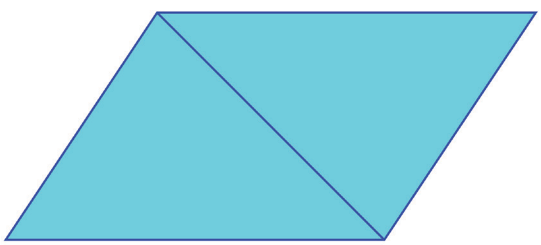
We have seen that several parallelograms can be drawn with the same lengths of sides; and they may not be of the same area.

What all measures of a parallelogram determine its area?

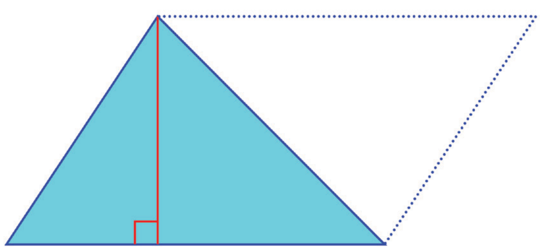
See this parallelogram:



To compute the area, let's first draw a diagonal to split it into two triangles:



What all measures are needed to compute the area of the left triangle?

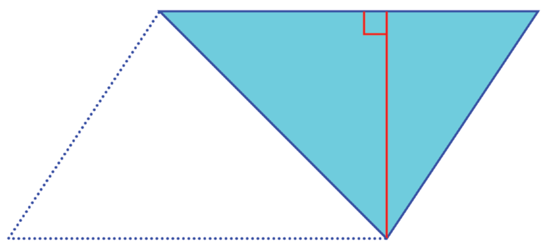


Its area is half the product of the length of the bottom side and the height from it to the top vertex.

The bottom side of the triangle is the bottom side of the parallelogram itself. What is the height to the top vertex?

Isn't it the distance between the top and bottom parallel sides of the parallelogram?

Now let's look at the triangle on the right:

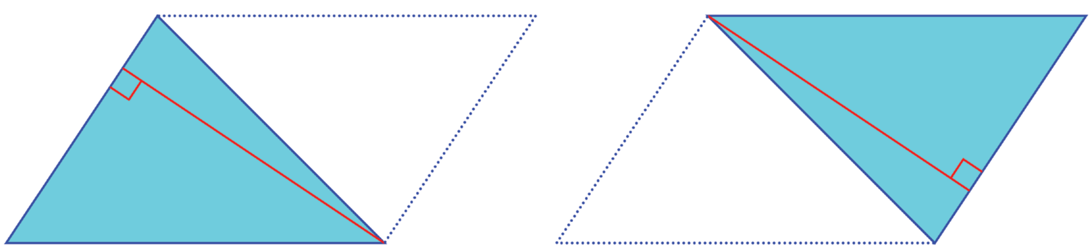


Its area is half the product of the length of the top side of the parallelogram and the distance between the top and bottom sides.

Thus both triangles have the same area. The area of the parallelogram is the sum of the areas of these two triangles; that is, twice the area of one of these triangles. What is it?

The product of the length of the bottom side (or the top side) of the parallelogram and the distance between the top and bottom sides.

What if we use the left and right sides of the triangles instead of the top and bottom sides?



What is the area of each triangle?

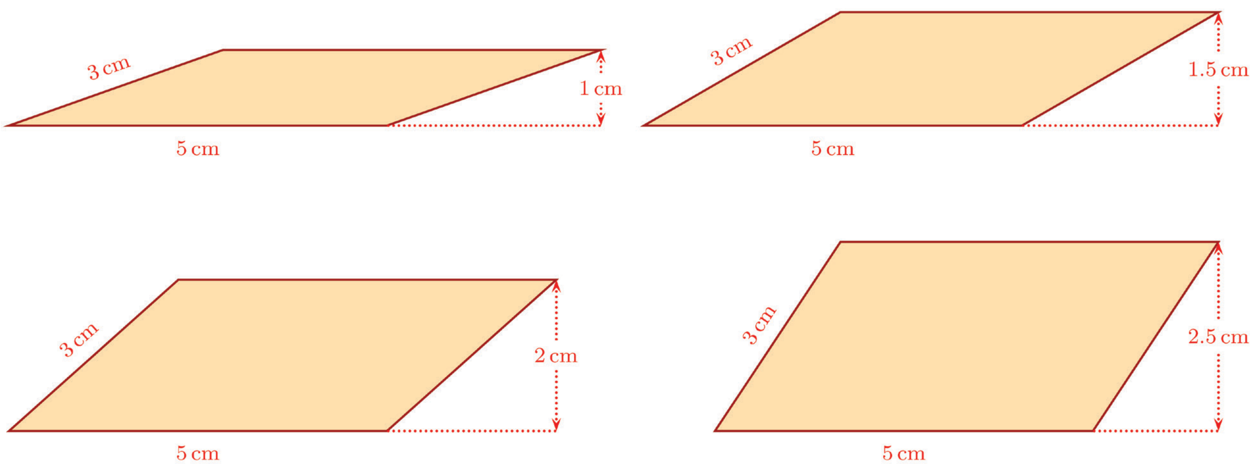
And the area of the parallelogram?

The product of the length of the left (or right) side and the distance between the left and right sides, isn't it?

So what can we say in general about the area of a parallelogram?

The area of a parallelogram is the product of the length of one side and its distance to the opposite side

Now see these pictures:



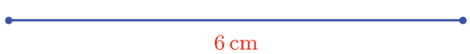
All of them have the lengths of sides 5 centimetres and 3 centimetres. Calculate the areas of each.

Can we increase the area further? What is the maximum possible area?

What is special about the parallelogram of maximum area with these lengths for the sides?

Now a question: how do we draw a parallelogram of sides 6 centimetres and 4 centimetres with area 18 square centimetres?

First let's draw a horizontal line of length 6 centimetres as the bottom side of the parallelogram.



The two bottom vertices of the parallelogram are the endpoints of these lines

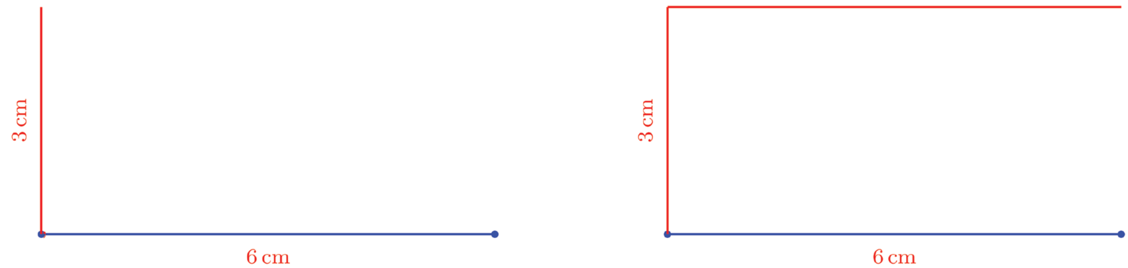
What about the two top vertices?

What can we say about the top side?

The area is to be 18 square centimetres. So, how high must be the top side from the bottom side?

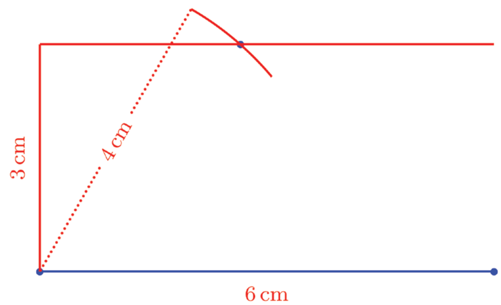
Draw a parallelogram ABCD of sides 5 centimetres and 3 centimetres in GeoGebra, as we did in the section Unchanging sides. Mark its area using the Area tool. Change the position of C. Does the area change? When is the area maximum? And minimum?

This means if we draw the perpendicular of height 3 centimetres from one end of the line we have drawn, and draw the line through its top end, parallel to the first line, then the top side of the parallelogram is a part of this line:

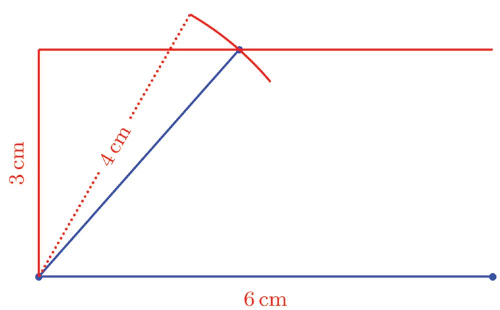


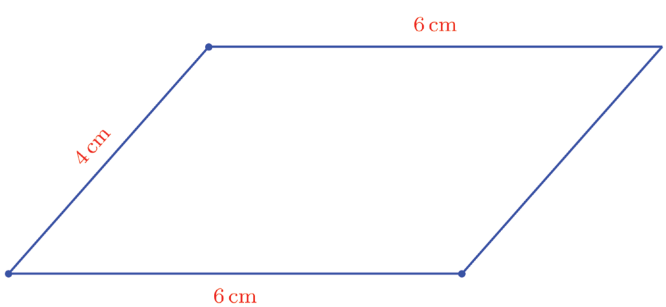
The top left vertex of the parallelogram is somewhere on the top line. The length of the left side of the parallelogram should be 4 centimetres. So, the top left vertex is at a distance of 4 centimetres from the bottom left vertex; and it should be on the top line.

How do we mark it?



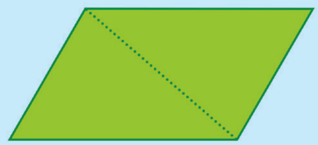
Now can't we complete the parallelogram?



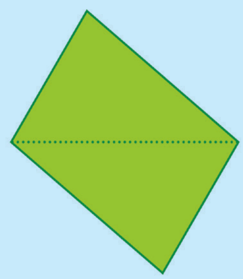


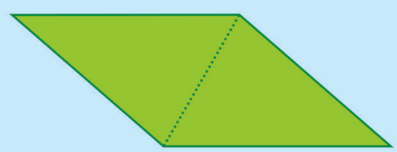
Unchanging area

See this parallelogram:



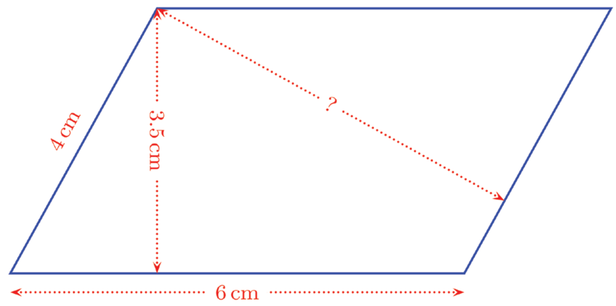
Cutting this along the diagonal, the parallel sides can be joined together to coincide in different ways to make new parallelograms:





How are the sides and a diagonal of these parallelograms related to the sides and a diagonal of the original parallelogram? What if we cut along the other diagonal and rejoin like this?

Another problem. See this parallelogram:



What is the distance between its left and right sides?

Since the length of bottom side is 6 centimetres and the distance between the top and bottom sides is 3.5 centimetres, the area of the parallelogram is $6 \times 3.5 = 21$ square centimetres.

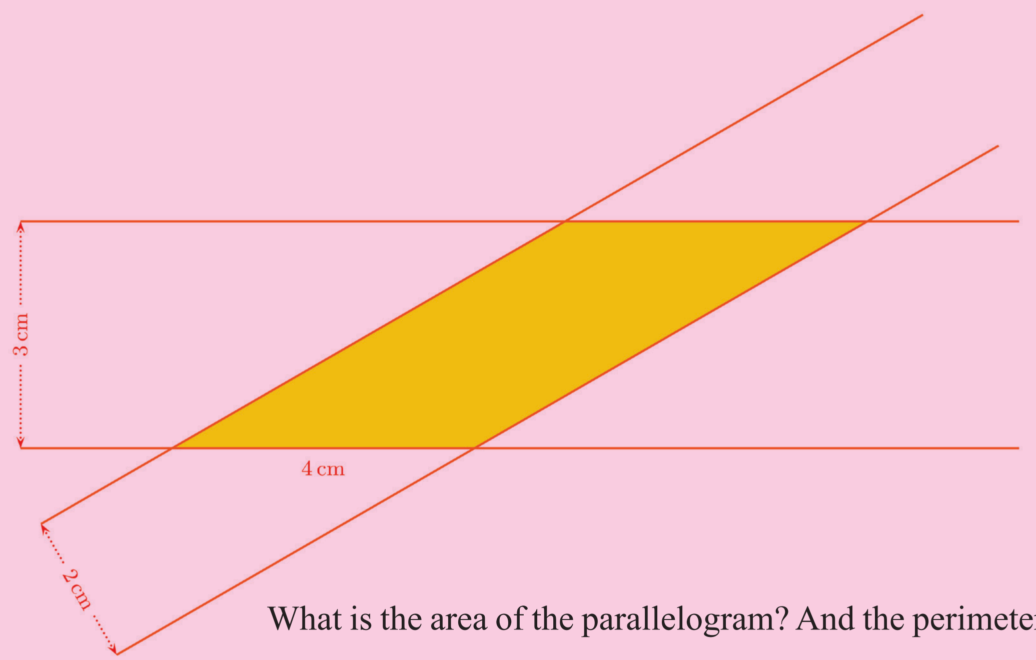
Since the length of the left side is 4 centimetres, the area is also got by multiplying the distance between the left and right sides by 4.

So, the distance between the left and right sides is $21 \div 4 = 5.25$ centimetres.

(1) Draw a parallelogram with lengths of sides 5 centimetres and 6 centimetres, and area 25 square centimetres.

(2) Draw a parallelogram with area 25 square centimetres and perimeter 24 centimetres.

(3) The picture shows the parallelogram formed by the intersection of two pairs of parallel lines:



What is the area of the parallelogram? And the perimeter?

(4) Two sides of a parallelogram are 12 centimetres and 10 centimetres, and the distance between the shorter sides is 6 centimetres.

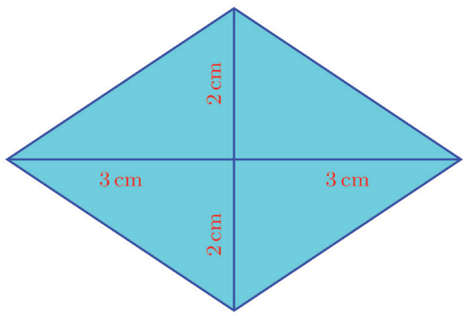
(i) What is the area of the parallelogram?

(ii) What is the distance between the longer sides?

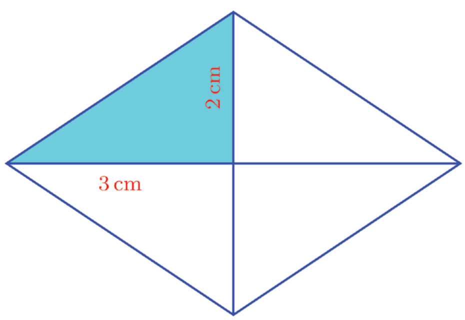
Rhombuses

To draw a rhombus we need only know the lengths of its diagonals (The section **Rhombuses** of the lesson **Bisectors**). How do we calculate the area of such a rhombus?

For example, let's look at a rhombus with lengths of diagonals 6 centimetres and 4 centimetres:



This is made up of four equal right triangles, isn't it? What is the area of one triangle?



$$\frac{1}{2} \times 3 \times 2 = 3 \text{ sq. cm}$$

All four triangles have the same area, right? So, the area of the rhombus is

$$4 \times 3 = 12 \text{ sq. cm}$$

We can calculate the area of any rhombus like this:

(i) A rhombus is four equal right triangles joined together.

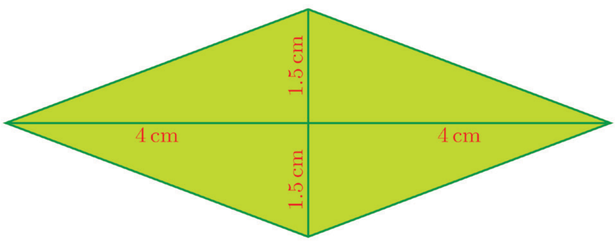
(ii) The perpendicular sides of each right triangle are half the diagonals of the rhombus

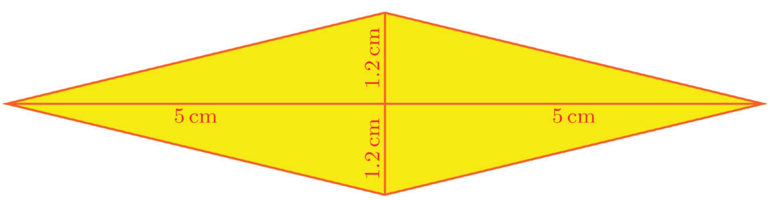
(iii) The area of a right triangle is half the product of half the diagonals; that is, one-eighth the product of the diagonals.

(iv) The total area of all the four right triangles is half the product of the diagonals

The area of any rhombus is half the product of the diagonals

So, we can draw a rhombus of area 12 square metres with diagonals of different lengths.





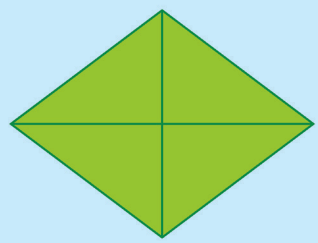
A square is a rhombus with equal diagonals, isn't it? So, in the case of a square, the product of the diagonals is the square of the diagonal.

Thus what can we say about the area of a square?

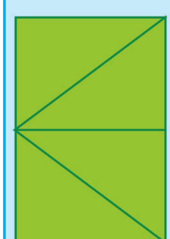
The area of any square is half the square of its diagonal

Rhombus and rectangle

Draw a rhombus and its diagonals:



Cut it along the diagonals to make four right triangles. They can be rearranged as a rectangle:



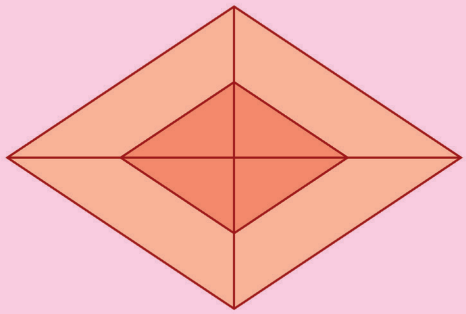
The area of this rectangle is same as the area of the rhombus, right?

What is the relation between the sides of this rectangle and the diagonals of the rhombus? So, what is the relation between the area of the rhombus and the length of its diagonals?

(1) Draw a square of area $4\frac{1}{2}$ square centimetres.

(2) Draw a rhombus of area 9 square centimetres, which is not a square.

(3) The picture shows a quadrilateral drawn by joining the midpoints of the half the diagonals of a rhombus:



(i) Is this quadrilateral also a rhombus? Why?

(ii) The area of the small quadrilateral is 3 square centimetres. What is the area of the large rhombus?

(4) The sides of a rhombus are 10 centimetres long and one of its diagonals is 16 centimetres long

(i) What is the length of the other diagonal?

(ii) What is the area of the rhombus?

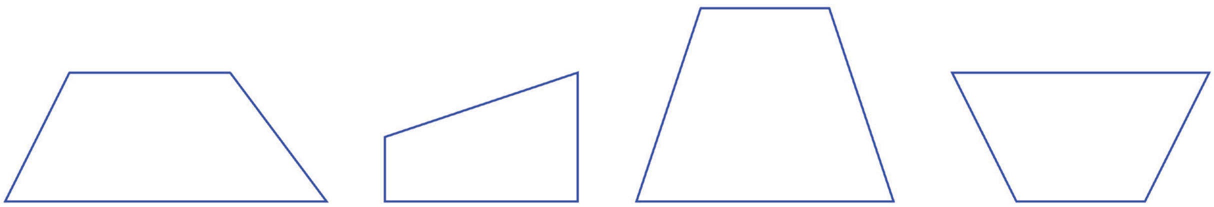
(iii) What is the distance between the opposite sides?

12

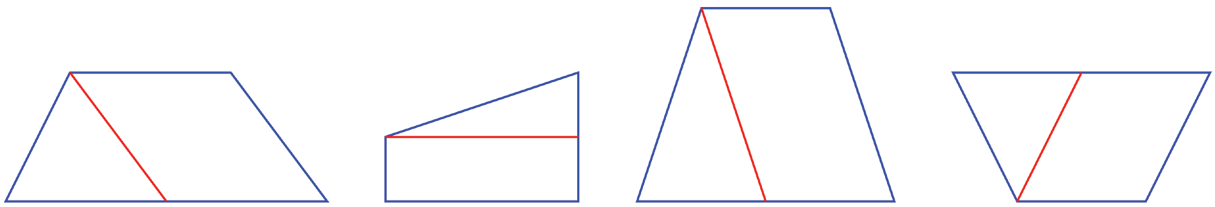
Trapeziums

Sides and angles

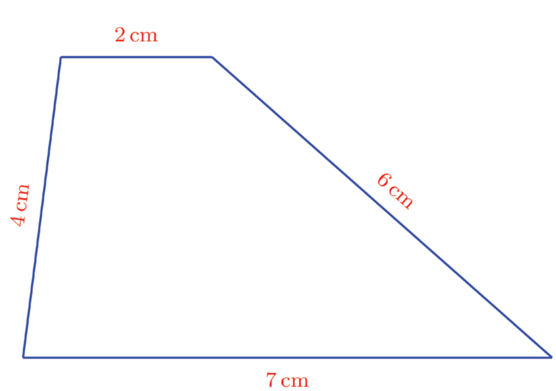
Parallelograms are quadrilaterals with both pairs of opposite sides parallel. A quadrilateral in which only one pair of opposite sides is parallel is called a **trapezium**:



By drawing within a trapezium, a line parallel to one of the non-parallel pair, we can split it into a parallelogram and a triangle:

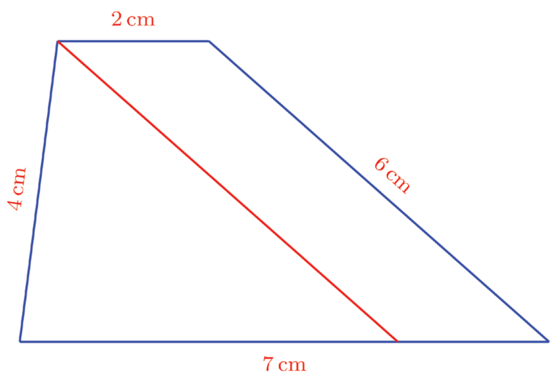


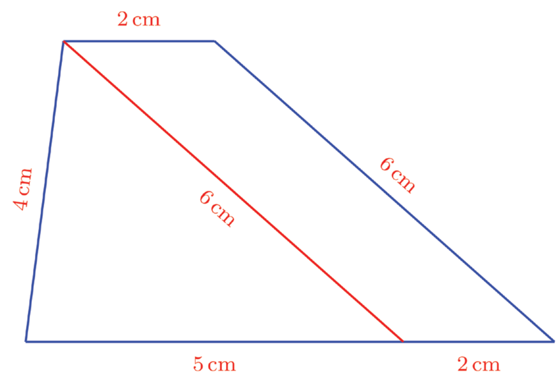
On the other hand, we can make a trapezium by combining a parallelogram with a suitable triangle. For example, let's see how we can draw a trapezium with sides specified as in the picture:



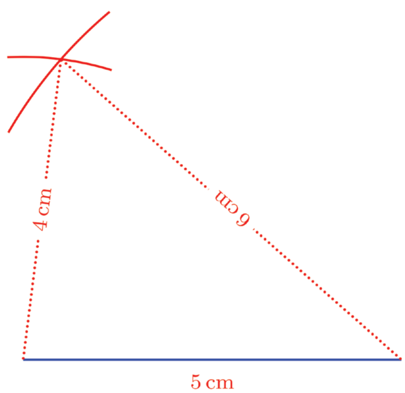
How about drawing a line parallel to the right side to split it into a parallelogram and a triangle?

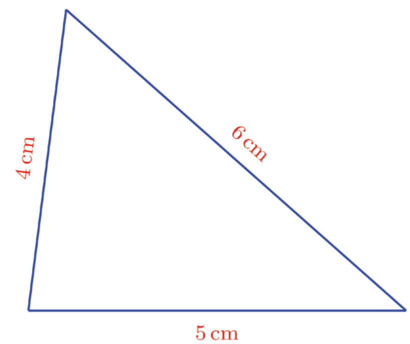
What are the lengths of sides of the triangle?



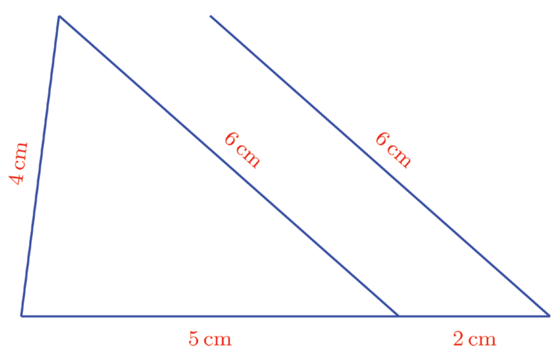


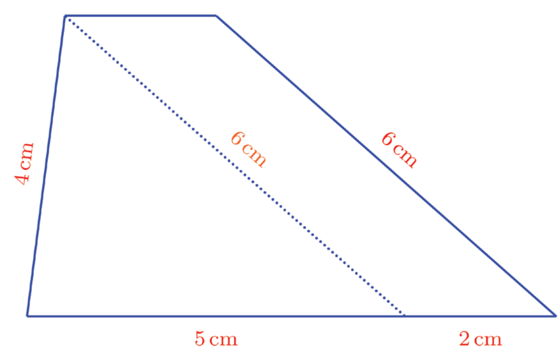
So, to draw this trapezium, let's draw the triangle first:





Next, extend the bottom side by 2 centimetres to the right and draw a line parallel to the right side. Mark 6 centimetres on this line and join its top end to the top vertex of the triangle to complete the trapezium:





Is the top side of the quadrilateral parallel to the bottom side, and its length 2 centimetres? Why?

Now instead of the lengths of all sides, if the lengths of the parallel sides and the angles which the other two sides make with one of these are specified, can we draw a trapezium according to these measures?

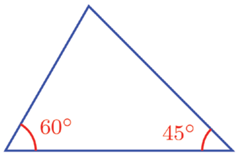
For example, see this trapezium:

Try to draw in GeoGebra, this trapezium with lengths and angles specified. The tools to use are Angle with Given Size, Line, Intersect, Parallel Line, Polygon.

As before we can split this into a triangle and a parallelogram:

We know one side of the triangle and one of the angles on it. To draw the triangle, we need to know the other angle also. How much is it?

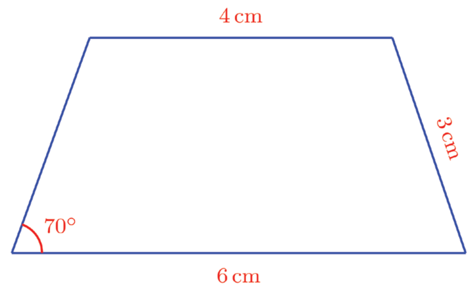
So, the triangle can be drawn like this:



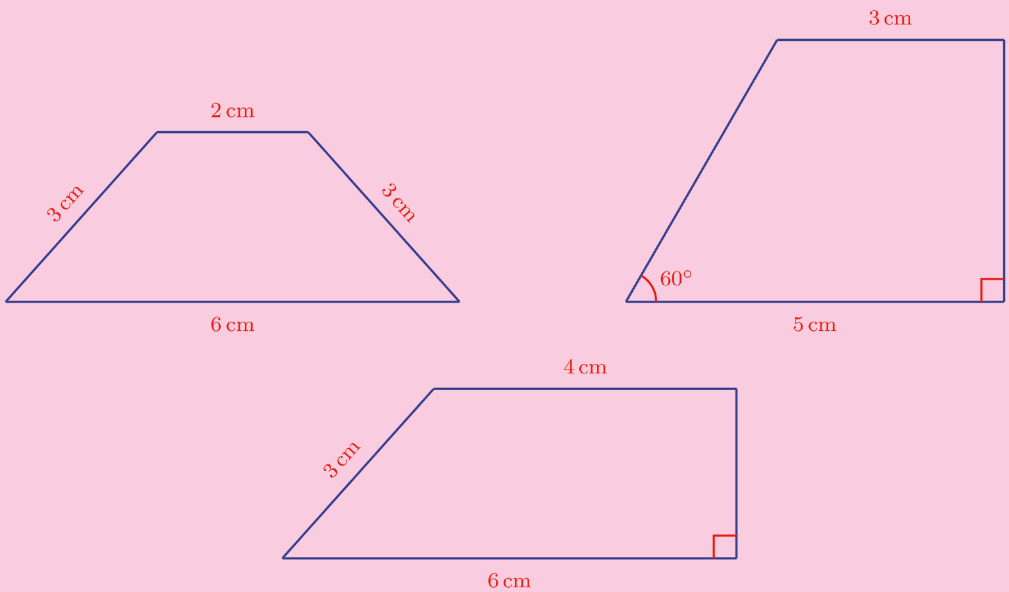
Next, extend the bottom side to the right by 5 centimetres and draw a line at an angle of 45°. Then draw the line parallel to the bottom side through the top vertex of the triangle to meet the slanted line on the right to complete the trapezium:

Why is the length of the top side 5 centimetres?

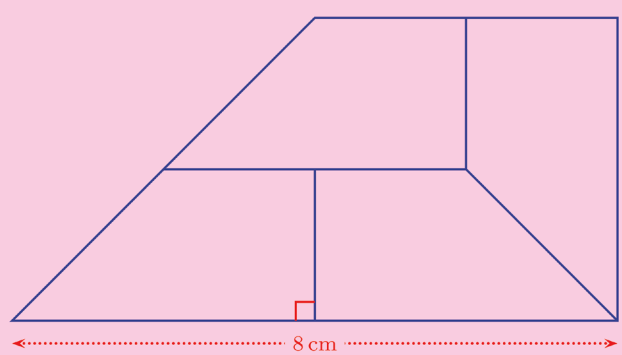
Can you draw the trapezium below?



(1) Draw the trapeziums below:



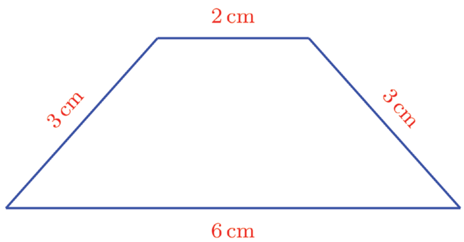
(2) The picture below shows four equal trapeziums joined together:



Draw this picture

Isosceles trapeziums

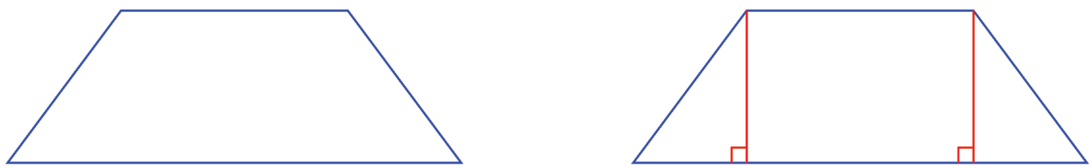
Have another look at a trapezium drawn earlier:



What is its peculiarity?

A trapezium in which the non-parallel sides have the same length is called an **isosceles trapezium.**

We have seen that any trapezium can be split into a parallelogram and a triangle. An isosceles trapezium can be split in a different way:



A rectangle and two triangles. What can we say about the triangles?

- Both are right triangles
- Hypotenuses are of the same length (why?)
- Vertical sides are of the same length (why?)

So, by Pythagoras Theorem, the third (horizontal) sides also have the same length.

Since all three sides are of the same length, the angles of the triangles are also equal.

Look at the marked angles in the first picture below:

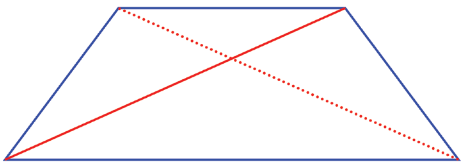
They are angles opposite to equal sides of the triangles as seen earlier. So they are equal.

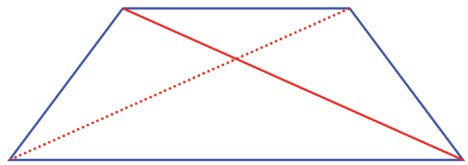
Thus, the angles marked in the second picture are also equal (How so?).

The parallel sides of a trapezium are generally called their bases. So these pairs of angles may be called base angles of the trapezium.

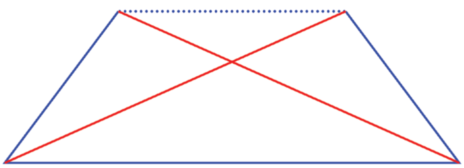
The base angles of an isosceles trapezium are equal

Let's look at another thing. Each diagonal of this isosceles trapezium divides it into two triangles:

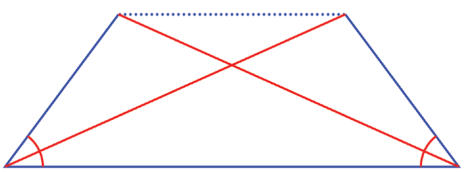




Let's compare the left and right triangles at the bottom:



The bottom side of both is the bottom side of the trapezium itself. Another pair of (blue) sides are equal, being the non-parallel sides of the isosceles trapezium. The angle between the bottom side and these sides are base angles of the isosceles trapezium and so are equal:

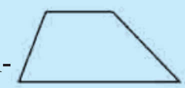


Since two sides and the angle between them are equal, the third (red) sides of the triangles are also equal. These sides are the diagonals of the trapezium. So, what can we say in general?

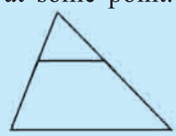
The diagonals of any isosceles trapezium are equal

Trapezium and triangle

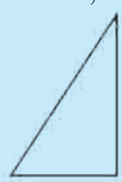
Draw a trapezium:



If we extend its non-parallel opposite sides, they will meet at some point. Then we have a triangle:



On the other hand, let's start with a triangle:



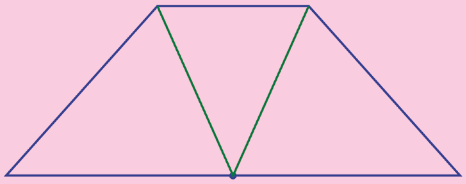
Draw a line within it, parallel to one of its sides.

If we now erase the two lines from the top, we get a trapezium, don't we?

If we start with an isosceles trapezium and extend the sides as above, what kind of triangle do we get?

Doing it in reverse, suppose we start with an isosceles triangle and cut off a part as above, what kind of trapezium do we get?

(1) The picture shows an isosceles trapezium with the midpoint of one base joined to the end points of the other base:

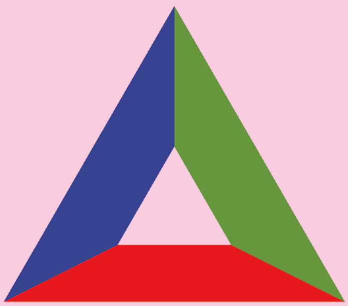


Prove that these lines are of the same length.

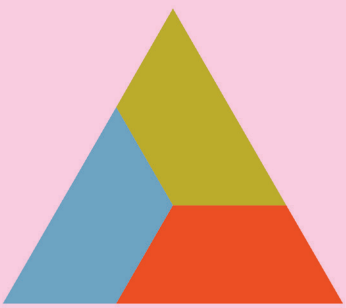
(2) Prove that the line joining the midpoints of the bases of an isosceles trapezium is perpendicular to both the bases.

(3) Draw the pictures below:

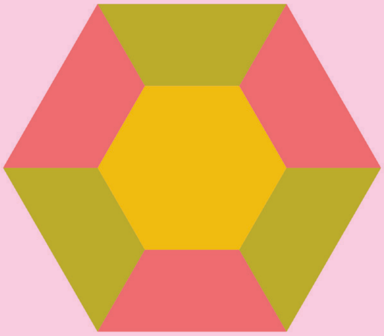
(i) Three equal isosceles trapeziums:



(ii) Three other equal isosceles trapeziums:



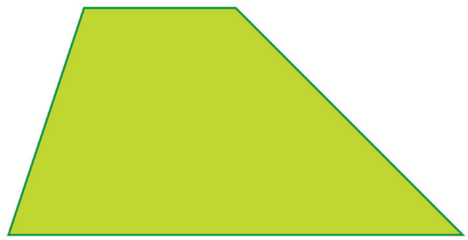
(iii) Six equal isosceles trapeziums:



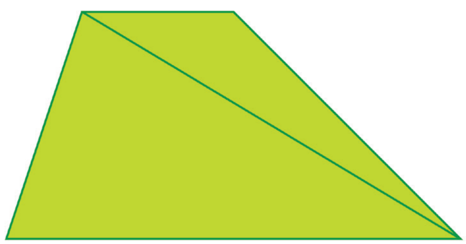
Draw these pictures in GeoGebra and colour them. After drawing an isosceles trapezium with specified measures, if we select Reflect about Line and click on the trapezium and one of its sides, we get a copy of the trapezium attached to the side.

Area

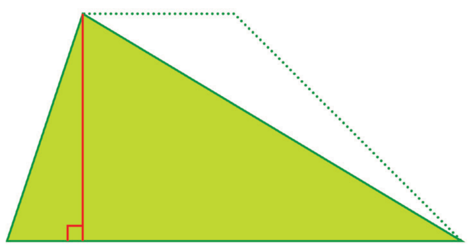
See the trapezium below. What all measures do we need to calculate its area?



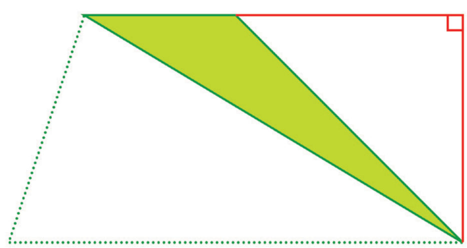
As we did in the case of a parallelogram, let's draw a diagonal to split it into two triangles:



What all measures do we need to calculate the area of the larger triangle?

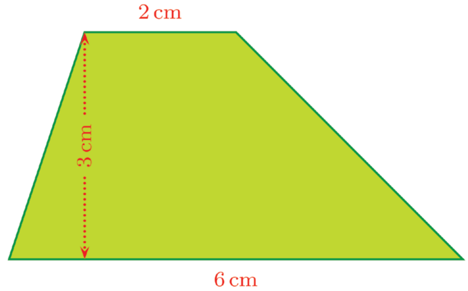


Its area is half the product of the length of the bottom side and the height from this side to the top vertex. That is, half the product of the length of the bottom side of the trapezium and the distance between the top and bottom sides.



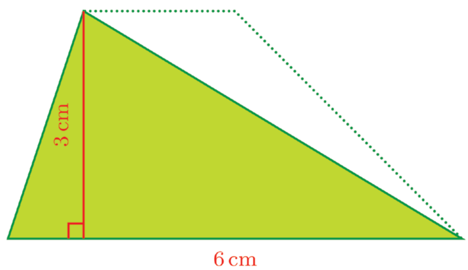
What about the smaller triangle?

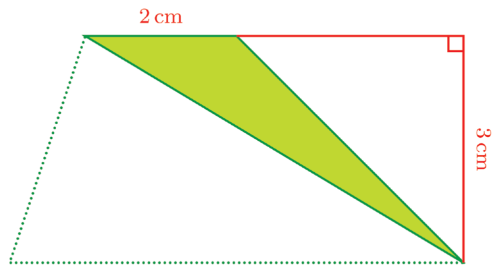
Its area is half the product of the length of the top side and the height from bottom vertex to this side extended. That is, half the product of the length of the top side of the trapezium and the distance between the top and bottom sides.



So to calculate the area of the trapezium, we need only the lengths of the top and bottom sides and the distance between them, right?

What are the area of the triangles got by drawing a diagonal as before?





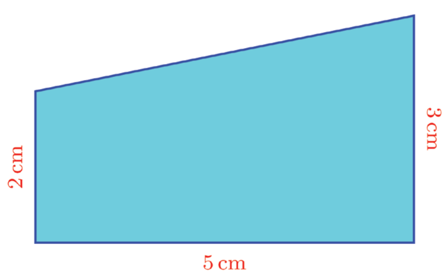
- Area of the triangle on the left is $\frac{1}{2} \times 6 \times 3 = 9$ square centimetres
- Area of the triangle on the right is $\frac{1}{2} \times 2 \times 3 = 3$ square centimetres
- Area of the trapezium $9 + 3 = 12$ square centimetres

We can write the computation of the area of the trapezium like this also:

$$\begin{aligned}\left(\frac{1}{2}\times 6\times 3\right)+\left(\frac{1}{2}\times 2\times 3\right) &= \frac{1}{2} \times (6+2) \times 3 \\ &= \frac{1}{2} \times 8 \times 3 \\ &= 12 \text{ square centimetres}\end{aligned}$$

That is, we added the lengths 6 and 2 of the parallel sides of the trapezium, multiplied the sum by the distance 3 between these sides and found half this product.

So, can you say what the area of the trapezium below is?



Draw different trapeziums in GeoGebra using the Polygon tool (Using Grid, we can easily draw them). Mark the area, the lengths of the parallel sides and the distance between them. See how they are related.

In this trapezium, the lengths of the parallel sides are 2 centimetres and 3 centimetres, and the distance between them is 5 centimetres.

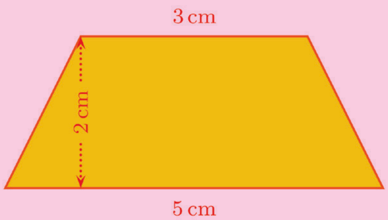
So the area is

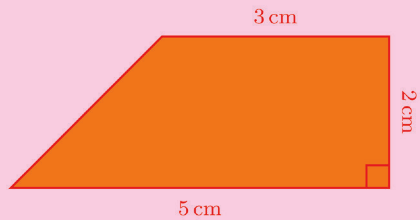
$$\frac{1}{2} \times (2 + 3) \times 5 = 12.5 \text{ square centimetres}$$

What can we say in general?

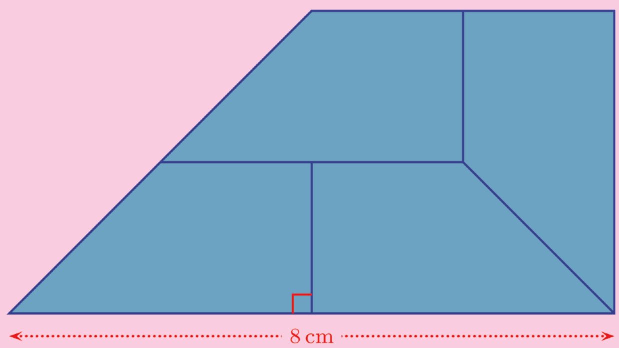
The area of a trapezium is half the product of the sum of the lengths of the parallel sides and the distance between them

(1) Calculate the area of the trapeziums shown below:



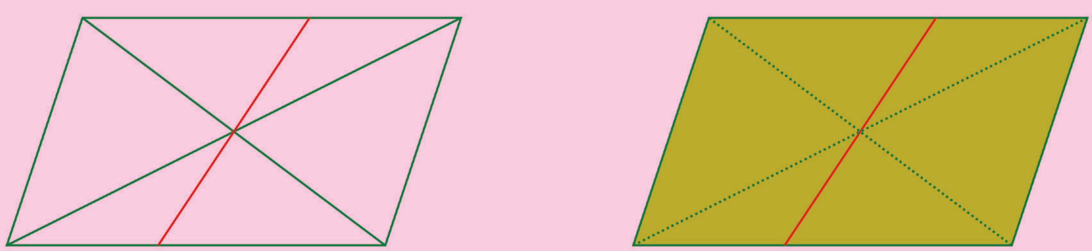


(2) The picture shows four equal trapeziums joined to form a large trapezium:



Calculate the area of the large trapezium

(3) The picture shows a line drawn through the point of intersection of the diagonals of a parallelogram:



Prove that this line splits the parallelogram into two trapeziums of the same area.

13

STATISTICS

Tabulation

There are 40 children in class 8A of the school. In a Health Club programme, their blood groups were identified and listed as below:

O+	B+	O+	AB+	AB–	B–
O+	AB–	AB+	AB+	B–	AB+
A+	O+	O+	O+	O+	A+
O–	A+	A+	O+	O+	O+
B+	B+	A+	A+	B+	O+
AB+	A+	B+	B+	O+	A+
B–	O+	O+	B+		

i) How many children have O– blood type?

ii) How many have B–?

iii) How many have O+?

iv) Which blood type do the largest number of children have?

v) Which blood type has the least number of children ?

To answer the first question, we need only count the O– types only. For the second, we count the B– types and for the third, the O+ types.

What about the fourth question?

We have to count each type separately, right?

Here, it is convenient to record this counting first:

Blood Group	Number
A+	8
B+	7
AB+	5
O+	14
B–	3
AB–	2
O–	1

Now can't you answer the last two questions easily?

Another problem.

The scores that children in a class got in a test are listed below:

8	7	6	3	8	8	7	7	6
7	9	7	6	8	7	2	6	7
10	6	7	3	9	5	4	5	4
4	4	5	8	10	8	8	9	7
7	6	8	8	7	4	5	9	8

i) What is the score obtained by the largest number of children?

ii) How many children got 8 or more?

iii) How many got less than 8?

iv) How many children got 10?

Let's make a table as before.

We must note how many times each score occurs.

The lowest score is 2 and the highest is 10.

Write the numbers from 2 to 10 in a column and check how many times each is repeated. We can use the method of tallies seen in class 5.

Score	Tally	Number of children
2	|	1
3	||	2
4	~~||||~~	5
5	||||	4
6	~~||||~~ |	6
7	~~||||~~ ~~||||~~ |	11
8	~~||||~~ ~~||||~~	10
9	||||	4
10	||	2
Total		45

Now it is easy to answer all questions above, just by looking at the table, isn't it?

The table shows how many times each score occurs, such as 2 once, 3 twice, 7 eleven times and so on. In tables of this kind, the number of occurrences is generally called frequency and the table itself is called a frequency table.

1) The number of members in 50 households of a village is listed below.

8	6	9	4	4	2	6	5	4	3
7	3	3	2	3	7	6	3	2	5
5	13	9	9	7	4	4	5	4	3
3	7	2	3	3	10	8	6	6	4
2	4	5	4	3	8	7	5	6	3

Make a frequency table and answer these questions:

i) How many households have just two members ?

ii) How many households have four or less ?

iii) How many households have ten or more ?

iv) Households of what size occur the most ?

2) There are 44 children in class 8B. The list shows how far they come from, in kilometres.

6	2	7	12	1	9	2	6
5	7	3	4	1	5	4	4
5	8	6	5	2	5	9	5
11	12	1	9	2	14	4	7
9	6	6	7	3	2	6	3
4	7	9	3				

Make a frequency table and answer these questions:

i) How many children are from exactly 1 kilometre away?

ii) How many are from more than 5 kilometres?

iii) How many are between 5 and 10 kilometres away?

iv) How many are from more than 10 kilometres?

3) The scores of 35 children in a test are given below:

15	10	18	11	19	16	15	17	14	18	13	15
17	16	15	14	15	17	14	15	13	16	11	11
16	20	13	12	10	16	17	13	12	14	12	

Make a frequency table and answer these questions:

i) How many children scored 20?

ii) How many children got scores between 10 and 15?

iii) How many scored less than 10?

iv) What is the score obtained by the largest number of children ?

Another form

The runs that a batsman got in 50 one-day cricket matches are given below.

50	0	49	60	100	68	27	48	15	65	101	45	2
52	25	18	29	53	72	90	32	81	28	104	35	49
2	60	87	71	38	102	35	71	68	20	10	30	55
47	21	35	12	20	11	27	43	38	40	48		

i) How many centuries did he get ?

ii) How many half-centuries?

iii) In how many games did he score less than 50?

Here the lowest score is zero and the highest is 104.

To make a table as we did so far, we would have to write all numbers from 0 to 104. But all such numbers are not really needed. Moreover from such a table, we don't get a general idea of the player's performance.

So, we do it in a slightly different way.

Instead of writing the actual runs in a column, we group them into classes as centuries (100 or more), half-centuries (50 - 99) and less than a half century (less than 50) and make a table:

Class	Tally	Number of games
0 - 49	卌 卌 卌 卌 卌 卌 |	31
50 - 99	卌 卌 卌	15
100 and more	||||	4
Total		50

Now from the table, can't we easily answer the questions asked?

Suppose we want to analyse the performance in a little more detail, to answer questions like these:

- In how many games did he score less than 10?
- In how many games did he score between 90 and 100?
- Between 40 and 50?

Then we would have to group the score into suitable classes and make a table.

We can group the scores as 0 to 9, 10 to 19, 20 to 29 and so on and count the number of games in each class.

On tables

To draw conclusions from a collection of data, we have to first put them in order. One method of such an arrangement is to classify them and form a table. A type of table used in statistics is the frequency table.

When we tabulate data like this, some information is lost. For example, when the entire data collected on incomes is presented as income groups and the number of people belonging to each group, we cannot find the actual income of each person from it.

But from such a table, we can get a general idea of how the various incomes are distributed among the people. Such a general view cannot be readily gained from the entire unorganised collection of data.

Class	Tally	Number of games
0 - 9	|||	3
10 - 19	~~||||~~	5
20 - 29	~~||||~~ |||	8
30 - 39	~~||||~~ ||	7
40 - 49	~~||||~~ |||	8
50 - 59	||||	4
60 - 69	~~||||~~	5
70 - 79	|||	3
80 - 89	||	2
90 - 99	|	1
100 - 109	||||	4
Total		**50**

Now we can easily answer the questions.

Let's look at another situation

The weights of the members of the school Health club are given below; in kilograms.

38	$37\frac{1}{2}$	$40\frac{1}{2}$	59	48	48	$37\frac{1}{2}$
58	50	$54\frac{1}{2}$	39	40	$40\frac{1}{2}$	49
32	43	45	53	37	44	51
$50\frac{1}{2}$	$32\frac{1}{2}$	46	55	36	$44\frac{1}{2}$	47
$42\frac{1}{2}$	33					

We want to make a frequency table.

Would classes like 30 – 34, 35 – 39, 40 – 44, 45 – 49 and so on, do?

In which class would we put $44\frac{1}{2}$ kilograms, for example.

We can take classes as 30 – 35, 35 – 40, 40 – 45 and so on $44\frac{1}{2}$ can be put in the class 40 – 45.

Classification methods

We classify and tabulate data for a concise presentation, from which it is easy to draw general conclusions. We have noted that some information is lost when we do this. This loss can be reduced by forming a large number of classes with small widths. But then the table will not be concise. On the other hand, if we form a few classes of large widths, the presentation would be compact, but the loss of information would be so great that no valid inferences could be drawn.

For example, when we tabulate incomes, suppose we divide the incomes into classes of width 1 rupee. All the collected information would be in the table; but there is no condensation of data. At the other extreme, if we consider the entire range of incomes as a single class, from the lowest income to the highest, then we have maximum condensation; but no general conclusions can be drawn from it.

But then, in which class would we put 40? 35 – 40 or 40 – 45? Usually, it is put in the class 40 – 45. Likewise, 45 is put in the 45 – 50 class.

Now we can make a frequency table:

Class	Tally	Frequency
30 - 35		
35 - 40		
40 - 45		
45 - 50		
50 - 55		
55 - 60		

1) Given below are the highest temperatures (in degree Celsius) one day in 40 towns. Make a frequency table.

41	23	32	40	25	30	38	47	40	39
26	31	37	32	36	41	30	25	27	30
29	40	38	36	43	37	28	27	32	36
38	36	33	32	28	27	23	26	28	31

2) The heights (in centimetres) of 45 people who took part in a physical fitness test are given below.

Make a frequency table.

160	145	168	156	168.4	170	163	177	143	175	169	154
163	176	160.3	164	150	168	166	148	154	159	164.5	
165	155	148.2	158	174	169	168	165	170	141	172.7	
179	167	171	159	167	171	165	171	167	162	171	

Height	Tally	Number
140 - 145		
145 - 150		
....		
....		
....		
....		

A new picture

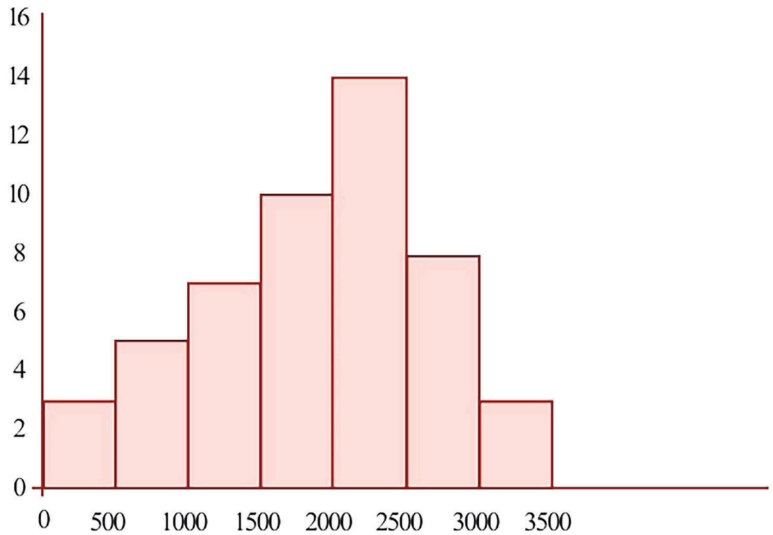
We have seen how numerical data can be pictorially represented as bar charts or pie diagrams.

Now let's see how the data given in a frequency table can be represented by a picture.

The table below gives the amount of water 50 households used in one day.

Amount of water (litres)	Number of households
0 - 500	3
500 - 1000	5
1000 - 1500	7
1500 - 2000	10
2000 - 2500	14
2500 - 3000	8
3000 - 3500	3
Total	**50**

See how this data is represented by a picture:



The classes are marked on the horizontal line and frequencies on the vertical line. The width of each rectangle shows the length of the class interval and its height shows the frequency. Such a picture is called a **histogram**.

1) The table shows the times 30 children took to complete a long distance race. Draw a histogram of this.

Time (min)	Number of children
10 - 13	3
13 - 16	5
16 - 19	12
19 - 22	8
22 - 25	2

2) The table shows the daily incomes of 60 households in a locality.

Daily income (Rupees)	Number of households
800 - 850	3
850 - 900	7
900 - 950	15
950 - 1000	20
1000 - 1050	9
1050 - 1100	6

Draw a histogram.

3) Details of rainfall in June and July are given in the table below. Draw a histogram.

Rainfall (mm)	Days
10 - 20	4
20 - 30	6
30 - 40	9
40 - 50	15
50 - 60	10
60 - 70	8
70 - 80	5
80 - 90	3
90 - 100	1

4) The time taken by 25 women and 23 men to complete a race are given in the table below. Draw separate histograms for men and women.

Time (sec)	Number	
	Women	Men
30 - 40	2	3
40 - 50	6	7
50 - 60	8	5
60 - 70	5	5
70 - 80	4	3

5) The weights of 45 children in a class are listed below.

41	31	48	34	75	39	45	41	55
52	40	57	43	61	47	64	56	47
41	59	46	67	45	64	48	52	58
53	64	59	43	50	62	54	68	59
69	57	57	53	52	56	61	55	69

Make a frequency table and draw a histogram.

Notes

Notes

CONSTITUTION OF INDIA

Part IV A

FUNDAMENTAL DUTIES OF CITIZENS

ARTICLE 51 A

Fundamental Duties- It shall be the duty of every citizen of India:

(a) to abide by the Constitution and respect its ideals and institutions, the National Flag and the National Anthem;

(b) to cherish and follow the noble ideals which inspired our national struggle for freedom;

(c) to uphold and protect the sovereignty, unity and integrity of India;

(d) to defend the country and render national service when called upon to do so;

(e) to promote harmony and the spirit of common brotherhood amongst all the people of India transcending religious, linguistic and regional or sectional diversities; to renounce practices derogatory to the dignity of women;

(f) to value and preserve the rich heritage of our composite culture;

(g) to protect and improve the natural environment including forests, lakes, rivers, wild life and to have compassion for living creatures;

(h) to develop the scientific temper, humanism and the spirit of inquiry and reform;

(i) to safeguard public property and to abjure violence;

(j) to strive towards excellence in all spheres of individual and collective activity so that the nation constantly rises to higher levels of endeavour and achievements;

(k) who is a parent or guardian to provide opportunities for education to his child or, as the case may be, ward between age of six and fourteen years.

CHILDREN'S RIGHTS

Dear Children,

Wouldn't you like to know about your rights? Awareness about your rights will inspire and motivate you to ensure your protection and participation, thereby making social justice a reality. You may know that a commission for child rights is functioning in our state called the ***Kerala State Commission for Protection of Child Rights.***

Let's see what your rights are:

- Right to freedom of speech and expression.
- Right to life and liberty.
- Right to maximum survival and development.
- Right to be respected and accepted regardless of caste, creed and colour.
- Right to protection and care against physical, mental and sexual abuse.
- Right to participation.
- Protection from child labour and hazardous work.
- Protection against child marriage.
- Right to know one's culture and live accordingly.
- Protection against neglect.
- Right to free and compulsory education.
- Right to learn, rest and leisure.
- Right to parental and societal care, and protection.

Major Responsibilities

- Protect school and public facilities.
- Observe punctuality in learning and activities of the school.
- Accept and respect school authorities, teachers, parents and fellow students.
- Readiness to accept and respect others regardless of caste, creed or colour.

Contact Address:

Kerala State Commission for Protection of Child Rights
'Sree Ganesh', T. C. 14/2036, Vanross Junction
Kerala University P. O., Thiruvananthapuram - 34, Phone : 0471 - 2326603
Email: childrights.cpcr@kerala.gov.in, rte.cpcr@kerala.gov.in
Website : www.kescpcr.kerala.gov.in

Child Helpline - 1098, Crime Stopper - 1090, Nirbhaya - 1800 425 1400

Kerala Police Helpline - 0471 - 3243000/44000/45000

Online ***R. T. E Monitoring : www.nireekshana.org.in***