MATHEMATICS

Part - 1

Standard
VIII

Government of Kerala
Department of General Education

Prepared by
State Council of Educational Research and Training (SCERT) Kerala

2025

THE NATIONAL ANTHEM

Jana-gana-mana adhinayaka, jaya he
Bharatha-bhagya-vidhata
Punjab-Sindh-Gujarat-Maratha
Dravida-Utkala-Banga
Vindhya-Himachala-Yamuna-Ganga
Uchchala-Jaladhi-taranga
Tava subha name jage,
Tava subha asisa mage,
Gahe tava jaya gatha
Jana-gana-mangala-dayaka jaya he
Bharatha-bhagya-vidhata
Jaya he, jaya he, jaya he,
Jaya jaya jaya, jaya he.

PLEDGE

India is my country. All Indians are my brothers and sisters.

I love my country, and I am proud of its rich and varied heritage. I shall always strive to be worthy of it.

I shall give my parents, teachers and all elders, respect and treat everyone with courtesy.

To my country and my people, I pledge my devotion. In their well-being and prosperity alone, lies my happiness.

MATHEMATICS

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Dear children,

We have seen in earlier classes how natural numbers and fractions are used to denote various measures. We have also seen how different combinations of measurements lead to mathematical operations on pure numbers and how the general principles of such operations are written using algebra. We also recognised special properties of geometric shapes such as rectangles, triangles and circles.

This book continues these studies. The method of solving practical problems on measures and the numbers denoting them using algebra, starts here. This method of converting physical problems to mathematical problems and solving them using algebraic techniques is used in almost all sciences. Because of this, it is an important aspect of mathematics education at all levels.

The experience of drawing triangles of specified measurements, gained in class seven, develops into the idea of equality of triangles in this book. In later classes it grows to the branch of mathematics called trigonometry; and this helps to measure distances and heights which cannot be directly measured.

We hope that this book will help you to understand the breadth of applications of mathematics and appreciate the beauty of its logic.

With love and regards,

Dr. Jayaprakash R.K.
Director
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CONTENT

CERTAIN ICONS ARE USED IN THIS TEXTBOOK FOR CONVENIENCE

 Let's do problems

 Project

 ICT possibilities

THE CONSTITUTION OF INDIA

PREAMBLE

WE, THE PEOPLE OF INDIA, having solemnly resolved to constitute India into a [1]**[SOVEREIGN SOCIALIST SECULAR DEMOCRATIC REPUBLIC]** and to secure to all its citizens :

JUSTICE, social, economic and political;

LIBERTY of thought, expression, belief, faith and worship;

EQUALITY of status and of opportunity; and to promote among them all

FRATERNITY assuring the dignity of the individual and the [2][unity and integrity of the Nation];

IN OUR CONSTITUENT ASSEMBLY this twenty-sixth day of November, 1949 do **HEREBY ADOPT, ENACT AND GIVE TO OURSELVES THIS CONSTITUTION.**

---

1. Subs. by the Constitution (Forty-second Amendment) Act, 1976, Sec.2, for "Sovereign Democratic Republic" (w.e.f. 3.1.1977)
2. Subs. by the Constitution (Forty-second Amendment) Act, 1976, Sec.2, for "Unity of the Nation" (w.e.f. 3.1.1977)

1
SQUARES

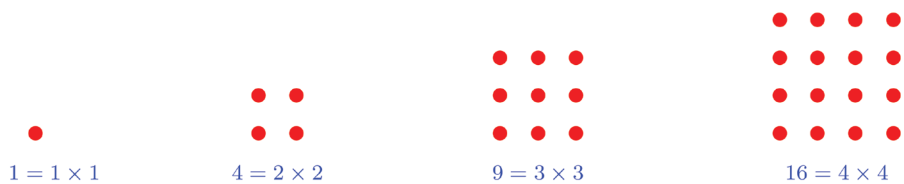
Perfect squares

Look at this pattern of numbers:

$$1, 4, 9, 16, \ldots$$

What is the speciality of these numbers?

Remember seeing such numbers in class 5 ? (The section, **Square numbers** of the lesson **Multiplication Methods**)



We saw in class 7 that these numbers can be written in another form (The lesson, **Repeated multiplication**):

$$\begin{aligned} 1 &= 1^2 \\ 4 &= 2^2 \\ 9 &= 3^2 \\ 16 &= 4^2 \\ 25 &= 5^2 \end{aligned}$$

....

And that they are called the square of 1, the square of 2, the square of 3 and so on (The section, **Areas of squares** of the lesson, **Squares and Right Triangles** in the Class 7 textbook).

The product of a fraction by itself is also called its square. For example,

$$\text{Square of } \frac{1}{2} = \left(\frac{1}{2}\right)^2 = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

The squares of the natural numbers 1, 2, 3, ... are called **perfect squares.**

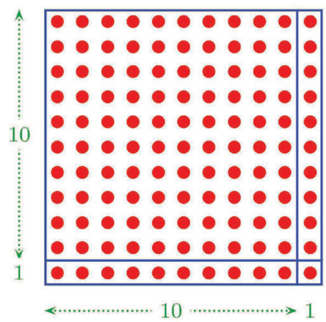
So, the numbers we saw at the beginning of the lesson are first four perfect squares.

It's quite easy to calculate the square of 10

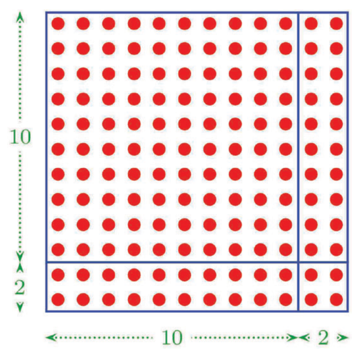
$$10^2 = 10 \times 10 = 100$$

What about the squares of the numbers after?

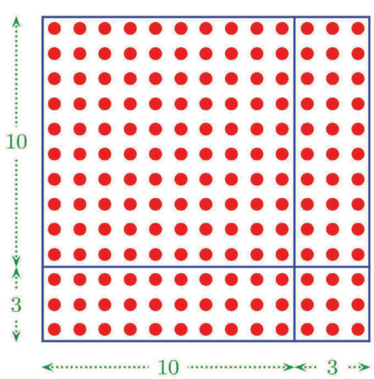
Remember how we multiplied two-digit numbers in class 5? (The section, **Rectangle multiplication** of the lesson, **Multiplication Methods**)



	10	1	
10	100	10	110
1	10	1	11
			121



	10	2	
10	100	20	120
2	20	4	24
			144



	10	3	
10	100	30	130
3	30	9	39
			169

Suppose in all these, we add the numbers in the cells diagonally, instead of horizontally and vertically?

	10	1
10	100	10
1	10	1

	10	2
10	100	20
2	20	4

	10	3
10	100	30
3	30	9

We can write these additions like this:

$$11 \times 11 = 100 + (10 + 10) + 1$$
$$12 \times 12 = 100 + (20 + 20) + 4$$
$$13 \times 13 = 100 + (30 + 30) + 9$$

We can also write them like this:

$$11^2 = 10^2 + (2 \times 10) + 1^2$$
$$12^2 = 10^2 + (2 \times 20) + 2^2$$
$$13^2 = 10^2 + (2 \times 30) + 3^2$$

Now can't you calculate the squares of 14 and 15 like this (without drawing cells)?

$$14^2 = 10^2 + (2 \times 40) + 4^2$$
$$= 100 + 80 + 16$$
$$= 196$$
$$15^2 = 10^2 + (2 \times 50) + 5^2$$
$$= 100 + 100 + 25$$
$$= 225$$

Try computing the squares of 16, 17, 18, 19 also like this.

Again, it's not difficult to calculate the square of 20

$$20^2 = 20 \times 20$$
$$= (2 \times 10) \times (2 \times 10)$$
$$= 2 \times 2 \times 10 \times 10$$
$$= 400$$

Let's calculate the squares of 21, 22, 23 as before, drawing cells:

	20	1
20	400	20
1	20	1

	20	2
20	400	40
2	40	4

	20	3
20	400	60
3	60	9

And we can write the additions like this:

$$21 \times 21 = 400 + (20 + 20) + 1 = 441$$

$$22 \times 22 = 400 + (40 + 40) + 4 = 484$$

$$23 \times 23 = 400 + (60 + 60) + 9 = 529$$

and then like this:

$$21^2 = 20^2 + (2 \times 20) + 1^2$$

$$22^2 = 20^2 + (2 \times 40) + 2^2$$

$$23^2 = 20^2 + (2 \times 60) + 3^2$$

How did we get the numbers 20, 40, 60 in the above?

$$21^2 = 20^2 + (2 \times 20 \times 1) + 1^2$$

$$22^2 = 20^2 + (2 \times 20 \times 2) + 2^2$$

$$23^2 = 20^2 + (2 \times 20 \times 3) + 3^2$$

Now can't we compute the square of 24, following this pattern?

$$\begin{aligned} 24^2 &= 20^2 + (2 \times 20 \times 4) + 4^2 \\ &= 400 + 160 + 16 \\ &= 576 \end{aligned}$$

Can we calculate the square of 36 like this?

	30	6
30	900	180
6	180	36

$$\begin{aligned} 36^2 &= 30^2 + (2 \times 30 \times 6) + 6^2 \\ &= 900 + 360 + 36 \\ &= 1296 \end{aligned}$$

Can't you now calculate the square of any two digit number like this, without drawing cells?

For example, let's take 79^2:

$$\begin{aligned} 79^2 &= 70^2 + (2 \times 70 \times 9) + 9^2 \\ &= 4900 + 1260 + 81 \\ &= 6241 \end{aligned}$$

Calculate the squares given below.

(i) 64^2 (ii) 35^2 (iii) 47^2 (iv) 53^2 (v) 88^2

Decimal squares

See this problem:

What is the area of a square of sides 3.7 metres?

Here, we must calculate 3.7×3.7

We have done such multiplications in class 7 (the section, **Decimal multiplication** of the lesson, **Decimal Methods**).

$$3.7^2 = \frac{37}{10} \times \frac{37}{10} = \frac{37^2}{100}$$

Now we can compute 37^2 as before:

$$\begin{aligned} 37^2 &= 30^2 + (2 \times 30 \times 7) + 7^2 \\ &= 900 + 420 + 49 \\ &= 1369 \end{aligned}$$

Thus

$$3.7^2 = \frac{37^2}{100} = \frac{1369}{100} = 13.69$$

and so, the area of the square is 13.69 square metres.

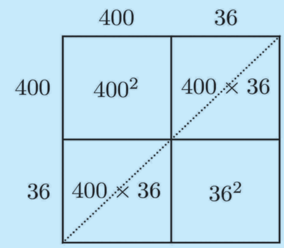
Three-digit numbers

How do we compute 436^2?

First let's do 36^2:

$$\begin{aligned} 36^2 &= 30^2 + (2 \times 30 \times 6) + 6^2 \\ &= 900 + 360 + 36 \\ &= 1296 \end{aligned}$$

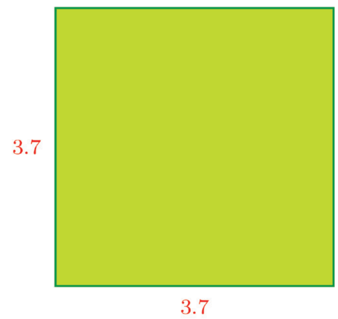
Now we can draw cells and compute 436^2



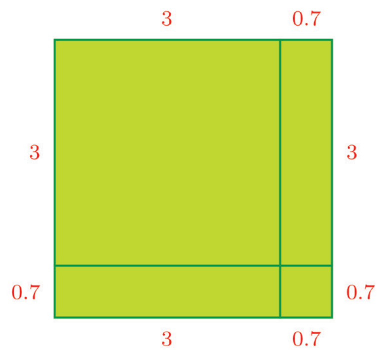
Thus

$$\begin{aligned} 436^2 &= 400^2 + (2 \times 400 \times 36) + 36^2 \\ &= 160000 + 28800 + 1296 \\ &= 190096 \end{aligned}$$

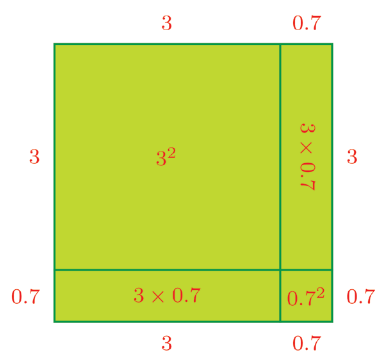
We can do this without converting the decimal to the fraction. For that, let's draw a picture:



And then split each side to 3 metres and 0.7 metre:



Then we compute the area of each of the four pieces separately:



Adding all these together we get the area of the square:

$$
\begin{aligned}
3.7^2 &= 9 + (2 \times 3 \times 0.7) + (0.7 \times 0.7) \\
&= 9 + 4.2 + 0.49 \\
&= 13.69 \text{ sq m}
\end{aligned}
$$

So, we can compute this square also by drawing cells as in the case of natural numbers:

	3	0.7
3	9	3×0.7
0.7	3×0.7	0.7×0.7

$$\begin{aligned} 3.7^2 &= 3^2 + (2 \times 3 \times 0.7) + (0.7)^2 \\ &= 9 + 4.2 + 0.49 \\ &= 13.69 \end{aligned}$$

Similarly we can compute 5.8^2 using only cells, without drawing a picture:

	5	0.8
5	25	5×0.8
0.8	5×0.8	0.8×0.8

$$\begin{aligned} 5.8^2 &= 5^2 + (2 \times 5 \times 0.8) + (0.8)^2 \\ &= 25 + 8 + 0.64 \\ &= 33.64 \end{aligned}$$

Now we can compute squares like this, without drawing cells even.

For example, let's compute 13.9^2

First we compute 13^2 as we did earlier:

$$\begin{aligned} 13^2 &= 10^2 + (2 \times 10 \times 3) + 3^2 \\ &= 100 + 60 + 9 \\ &= 169 \end{aligned}$$

Then we compute 13.9^2 as we did just now:

$$\begin{aligned} 13.9^2 &= 13^2 + (2 \times 13 \times 0.9) + (0.9)^2 \\ &= 169 + 23.4 + 0.81 \\ &= 193.21 \end{aligned}$$

(1) Find the squares of the numbers below:

(i) 2.3 (ii) 8.7 (iii) 10.1 (iv) 12.5 (v) 15.7

(1) Look at these computations:

1.5^2	2.5^2	3.5^2
$= 1^2 + (2 \times 1 \times 0.5) + (0.5)^2$	$= 2^2 + (2 \times 2 \times 0.5) + (0.5)^2$	$= 3^2 + (2 \times 3 \times 0.5) + (0.5)^2$
$= 1 + 1 + 0.25$	$= 4 + 2 + 0.25$	$= 9 + 3 + 0.25$
$= 2.25$	$= 6.25$	$= 12.25$

Can you calculate 4.5^2 like this?

Take some more numbers with decimal part 0.5 and calculate their squares. Can you find a simple method to calculate the squares of such numbers ?

(2) Look at these computations:

$$1.25^2 = 1^2 + (2 \times 1 \times 0.25) + (0.25)^2 = 1 + 0.5 + 0.0625 = 1.5625$$

$$2.25^2 = 2^2 + (2 \times 2 \times 0.25) + (0.25)^2 = 4 + 1 + 0.0625 = 5.0625$$

$$3.25^2 = 3^2 + (2 \times 3 \times 0.25) + (0.25)^2 = 9 + 1.5 + 0.0625 = 10.5625$$

$$4.25^2 = 4^2 + (2 \times 4 \times 0.25) + (0.25)^2 = 16 + 2 + 0.0625 = 18.0625$$

Take some more numbers with decimal part 0.25 and calculate their squares. Is there a general method to compute such squares ?

2

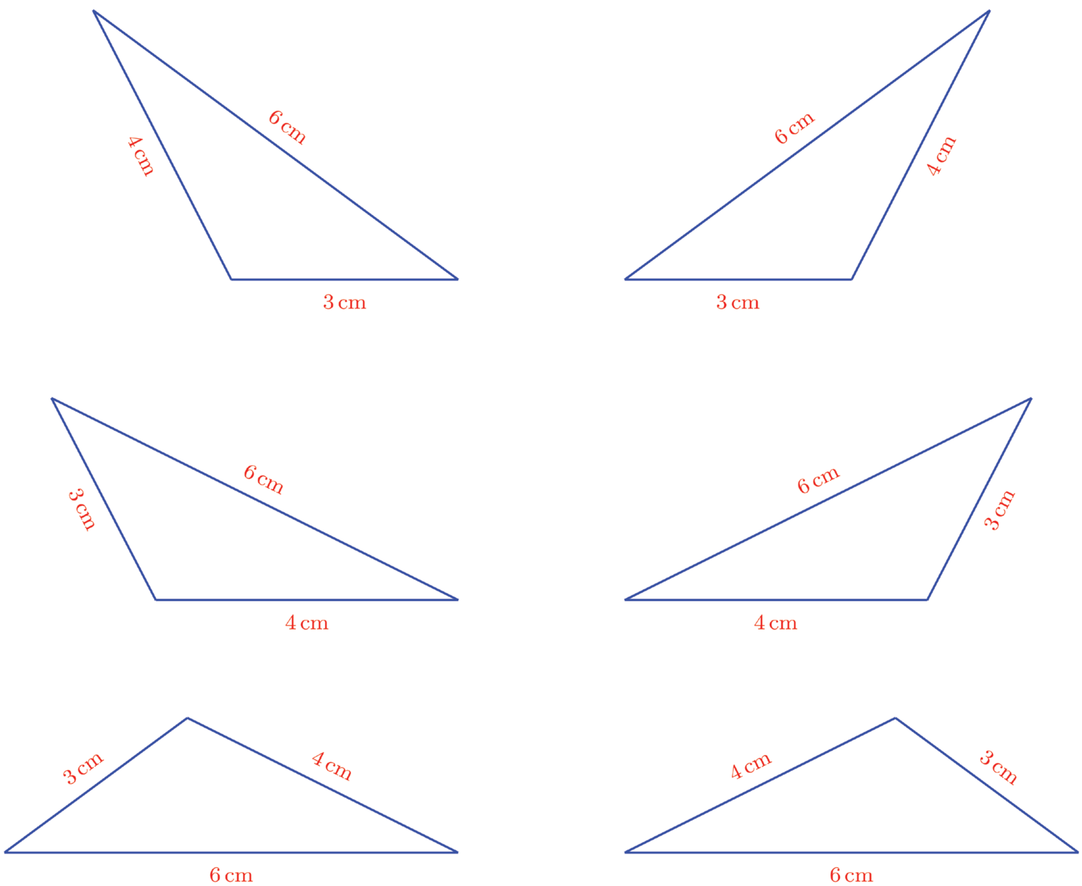
Equal Triangles

Sides and angles

We know how to draw a triangle if lengths of sides are specified.

For example, how do we draw a triangle with lengths of sides 3 centimetres, 4 centimetres and 6 centimetres ?

We have seen in class 7 that this can be drawn in several ways (the section **Lines and math** of the lesson, **Triangles**).



These triangles are the same, except that they are flipped and turned in various ways.

To make sure, cut out one of these triangles from a stiff piece of paper and place it over the others in different ways to see if they match.

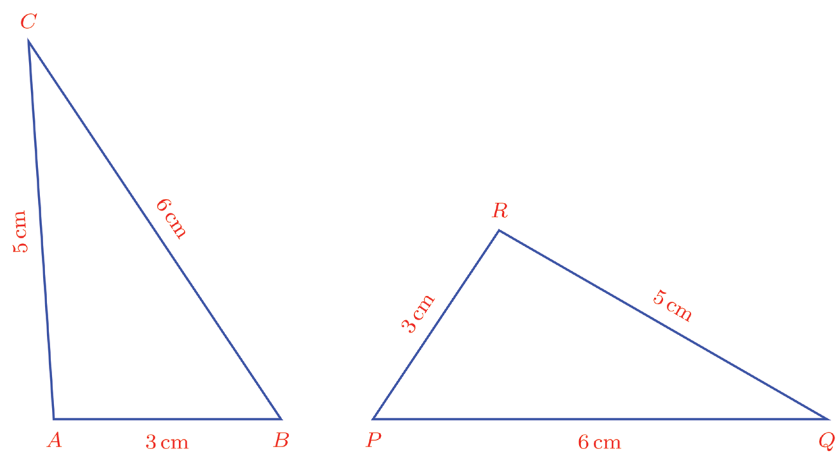
When they are made to coincide like this, not only the sides of the same length, but the angles also match, right ?

Take three other lengths for the sides to draw triangles like these and check. Aren't the angles the same ?

So, what can we say in general ?

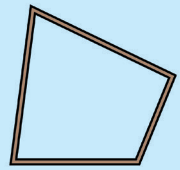
If the lengths of the sides of two triangles are the same, then their angles are also the same

Now look at these triangles:

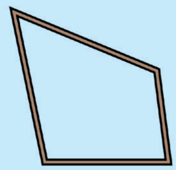


Changing quadrilaterals

Make a quadrilateral with four eerkkil bits:



If we shift the bits slightly, we can change the shape like this:



The lengths of the sides of both the quadrilaterals are the same; but aren't the angles different ?

Draw triangle *ABC* and mark a point *D* out side it. Select *Move around point tool* and click on the point *D* and the triangle and rotate the triangle around *D*. Do the sides or angles of the triangle change?

Since the lengths of the sides of the two triangles are the same, their angles are also the same.

That is each angle in triangle *ABC* is equal to one of the angles in triangle *PQR*.

But how do we find out which angle is equal to which?

The angles of the two triangles can be paired as the smallest angles, medium sized angles and the largest angles and these pairs are equal.

How do we know the order of the sizes of angles in each triangle?

We have seen in class 7 that the sizes of angles in any triangle is in the same order as the sizes of their opposite sides (The section, **Lines and math** of the lesson **Triangles**).

In the two triangles here,

- The longest side is 6 centimetres
- The medium side is 5 centimetres
- The shortest side is 3 centimetres

Thus in each triangle

- The largest angle is opposite the 6 centimetre side
- The medium angle is opposite the 5 centimetre side
- The smallest angle is opposite the 3 centimetre side

Now can't we say which angle in triangle ABC are equal to which angles in triangle PQR ?

Angles opposite the 6 centimetre sides $\angle A = \angle R$ (largest angles)

Angles opposite the 5 centimetre sides $\angle B = \angle P$ (medium angles)

Angles opposite the 3 centimetre sides $\angle C = \angle Q$ (smallest angles)

So, we can make our earlier statement a bit more precise:

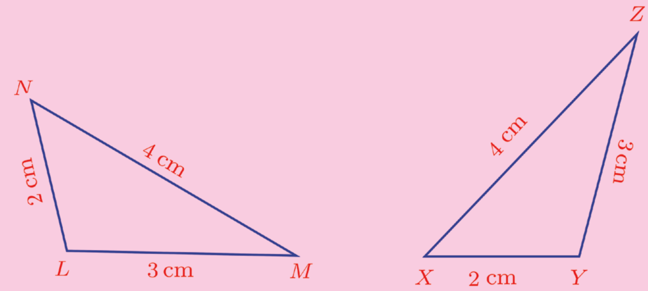
If the lengths of the sides of two triangles are the same, then the angles opposite to sides of equal length are also equal

Stability of triangles

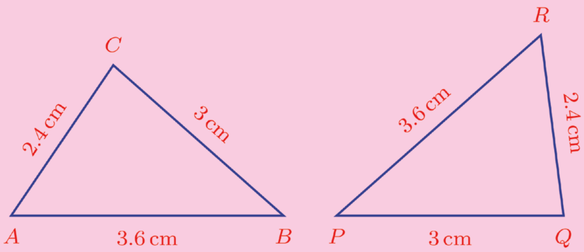
The angles of a quadrilateral can be changed without altering the length of the sides. So a quadrangular frame can tilt under force. But this wont happen in a triangular frame, since the angles of a triangle cannot be changed without altering the lengths of sides. It is because of this that triangular frameworks are used in the construction of bridges and towers.

(1) In each pair of triangles below, find the angles of the second triangle equal to the angles of the first triangle and write these pairs.

(i)

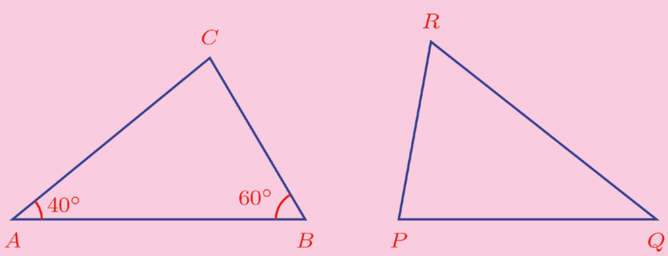


(ii)



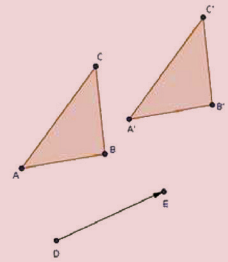
(2) In the triangles shown below,

$AB = QR \qquad BC = RP \qquad CA = PQ$



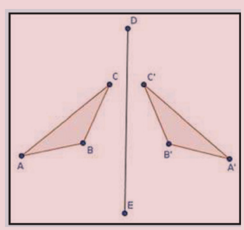
Calculate $\angle C$ of triangle ABC and all angles of triangle PQR

Draw triangle ABC in GeoGebra. Mark two points D and E. Select *Translate by Vector* and click on the triangle ABC and then the points D and E. We get a new triangle A'B'C'



What is the relation between the two triangles? Change the sides and angles of triangle ABC. Change the position of E. What happens when E coincides with D?

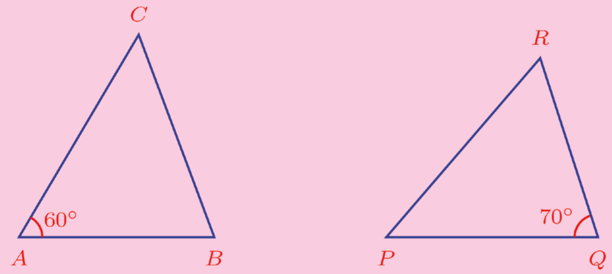
Next draw triangle ABC and line DE. Select *Reflect about Line* and click on the triangle and the line. We get another triangle A'B'C'.



What is the relation between the two triangles? Change the lengths of the sides of triangle ABC, the length and slant of the line DE. What do you see?

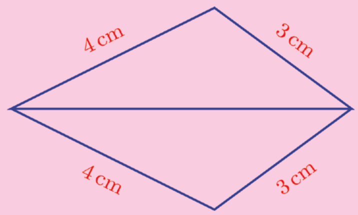
(3) In the triangles below,

$AB = QR \qquad BC = PQ \qquad CA = RP$



Calculate the other two angles of each triangle.

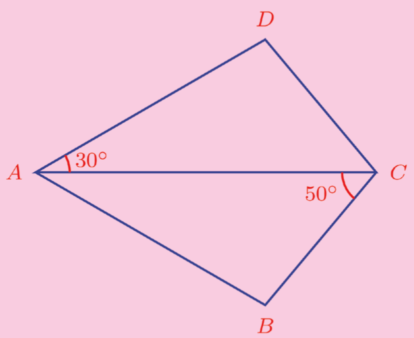
(4) The diagonal of a quadrilateral splits it into two triangles as shown below:



Do these triangles have the same angles? Why?

(5) In the quadrilateral $ABCD$ shown below,

$AB = AD, \qquad CB = CD$



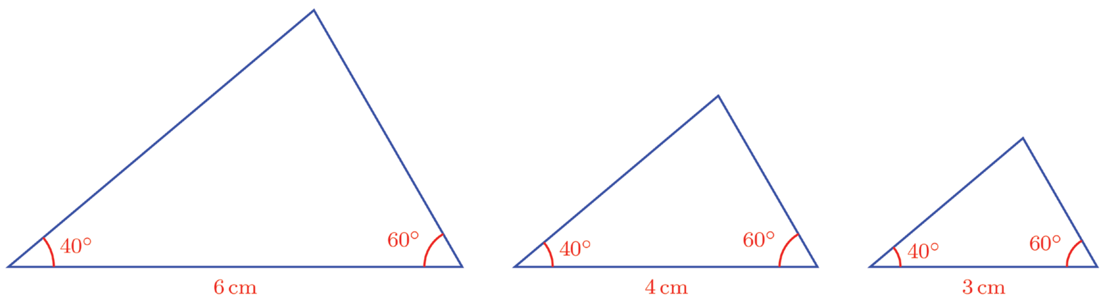
Calculate all the angles of the quadrilateral.

Two angles

We have seen that if the lengths of the sides of a triangle is same as the lengths of the sides of another triangle, then the two triangles have the same angles also.

On the other hand, if the angles of a triangle are the same as the angles of another triangle would the lengths of the sides be the same?

See these pictures:



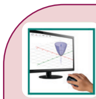
Haven't you drawn such triangles in class 7 ? (The section **Angle math** of the lesson **Triangles**).

In each of these triangles, the third angle is 80°.

What about the sides?

What do we see from this ?

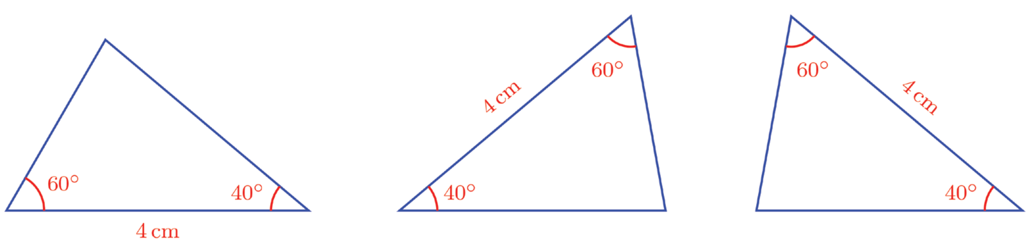
> Even if the angles of a triangle is same as the angles of another triangle, the lengths of their sides may not be the same.



In GeoGebra, make a slider a with Min:0 and Max:5. Draw a triangle with sides 4, 5, 6 and another with sides 4a, 5a, 6a. Look at the angles of these triangles (Select the *Angle tool* and click on a triangle to get its angles). Change the value of **a** using the slider. What happens ? What about the triangles when **a** is made 1 ?

The triangles above have the same two angles drawn at the ends of lines of different lengths.

Any one of them can be drawn in different ways:



In all these, one side and the angles at its ends are the same

Triangles of the same three measures can be drawn by changing the positions of the sides and angles; but all of them are just one triangle flipped or rotated, as seen in class 7.

In other words, the third angle and the lengths of the other two sides do not change in any of them.

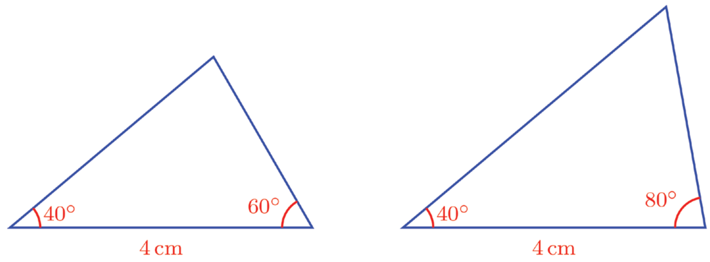
So, what can we say in general ?

If in two triangles , the length of one side and the two angles at its ends are the same , then their third angles and the lengths of the other two sides are also the same

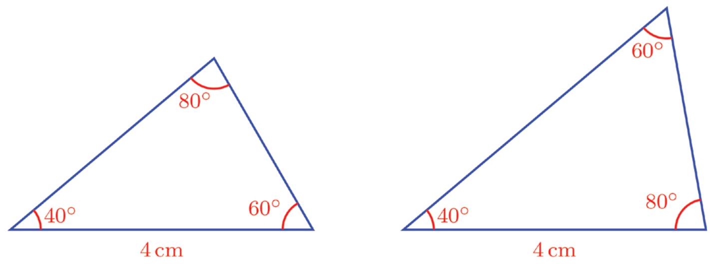
If we know two angles of a triangle, then we can calculate the third angle also, since the sum of all three angles of any triangle is 180°.

So if two angles of a triangle are the same as two angles of another triangle, then the third angles are also the same; if any side of one is equal to a side of the other, would the remaining two sides also be equal ?

Draw two triangles like this:



What are the third angles ?

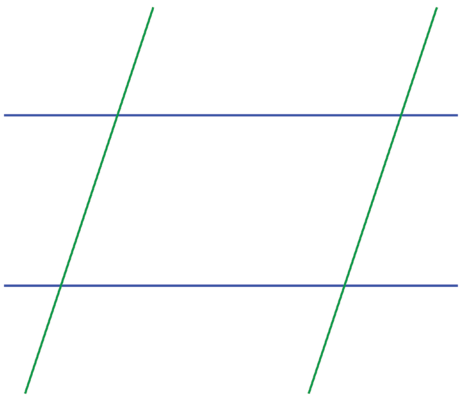


Both triangles have the same angles; and the lengths of the bottom sides are also the same; yet the remaining two sides are not the same, as we can see at a glance.

In two triangles, even if all the angles and one side are the same, the other two sides may not be the same.

Let's look at some application of the general result above.

A parallelogram is the figure formed by the intersection of two pairs of parallel lines:



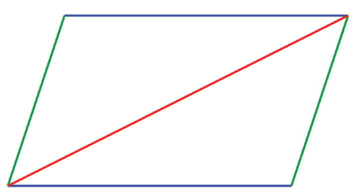
In other words , a parallelogram is a quadrilateral with each pair of opposite sides parallel.

Each pair of opposite sides of the parallelogram shown above also seems to be equal, doesn't it ? How can we be sure ?

One way of seeing that two lines are equal, is to check whether they are sides of a triangle with other measures equal.

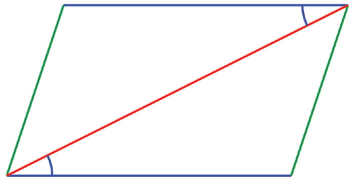
But the picture above does not have any triangle.

If we draw a diagonal of a quadrilateral, we do get two triangles, right ?

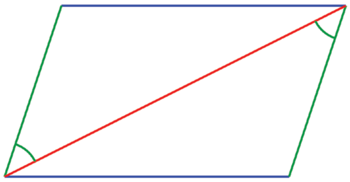


What can we say about the angles of these two triangles ?

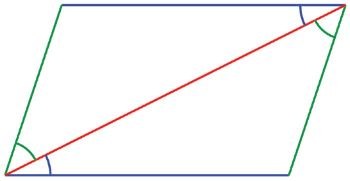
Since the blue lines in the picture are parallel, the angles which the red line makes with them are equal:



And since the green lines are parallel, the angles which the red line makes with them are also equal:



In other words, the red side and the two angles on it are the same for both the top and bottom triangles.



So, the opposite sides of the equal angles must also be equal.

Thus the opposite side of the parallelogram are equal.

We state this as a general result:

The opposite sides of any parallelogram are equal

From the reasoning above, don't you see that the opposite angles of the parallelogram are also equal.

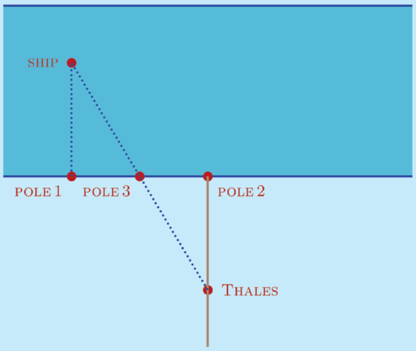
The opposite angles of any parallelogram are equal

Triangle technique

Thales was a philosopher and mathematician who lives in Greece during the sixth century BCE. Here's is a trick he is supposed to have used to calculate the distance to a ship anchored at sea from the shore.

First he stuck a long pole on the shore, directly in front of the ship. Then he stuck another pole on the shore, some distance away from the first one. A third pole he stuck exactly at the middle of the first two poles.

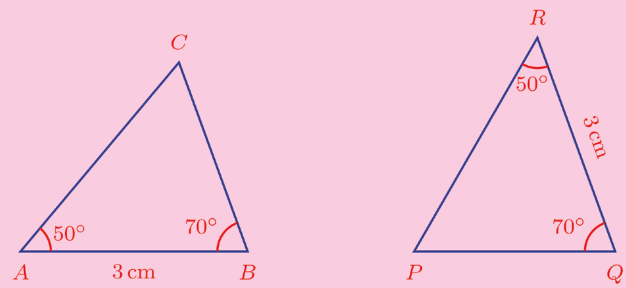
He then drew a line from the second pole, perpendicular to the shore. He walked backwards along this line, keeping the ship in sight. Just when the middle pole came between the ship and himself in the line of sight, he stopped and marked his position:



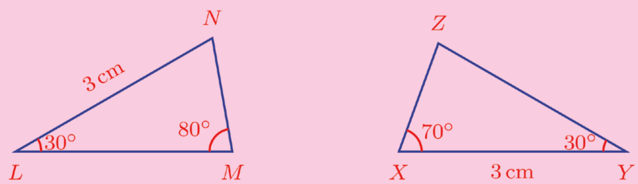
Now in the triangle on sea and the triangle on shore, the sides along the shore and the angles at their ends are the same (why ?). So, the distance from the shore to the ship is equal to the distance between the spot where he stopped and the second pole.

(1) In each pair of triangles below, find the sides of the triangle on the right equal to the sides of the triangle on the left:

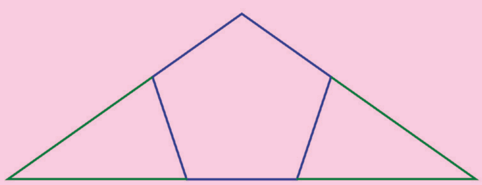
(i)



(ii)



(2) In the picture, the top two sides and the bottom side of a pentagon, with equal angles and equal sides, are extended to form a triangle:



(i) Are the sides of the small triangle on the left equal to the sides of the small triangle on the right ? Why ?

(ii) Are the left and right sides of the large triangle equal ? Why ?

(3) The sides of a triangle are equal to the sides of another triangle.

(i) Is the height from each side of one triangle to the opposite vertex equal to the height from the equal side of the other triangle to its opposite vertex ? Why ?

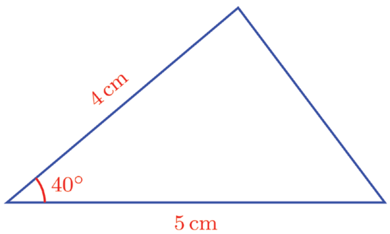
(ii) Are the areas of the two triangles equal ? Why ?

Two sides

To draw a triangle , it is enough to specify the lengths of all three sides; or the length of a side and the angles at its ends.

Can we draw a triangle if any other three measures are specified ?

How about the lengths of two sides and the angle between them ?

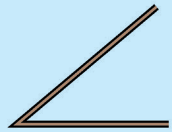


We have drawn such triangles in class 7 (the section, **Sides and angles** of the lesson **Triangles**).

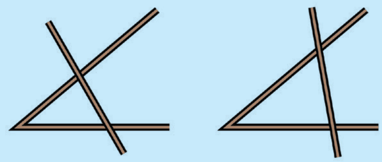
As we have done with other triangles, this can also be drawn in various ways by changing the positions of vertices and sides, which just amounts to turning and flipping the one shown above :

Determining a triangle

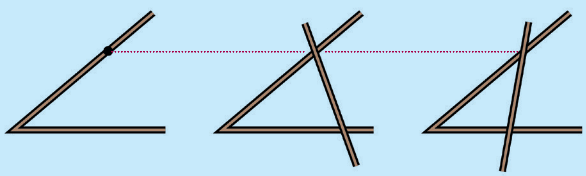
Make an angle by bending a long piece of eerkkil.



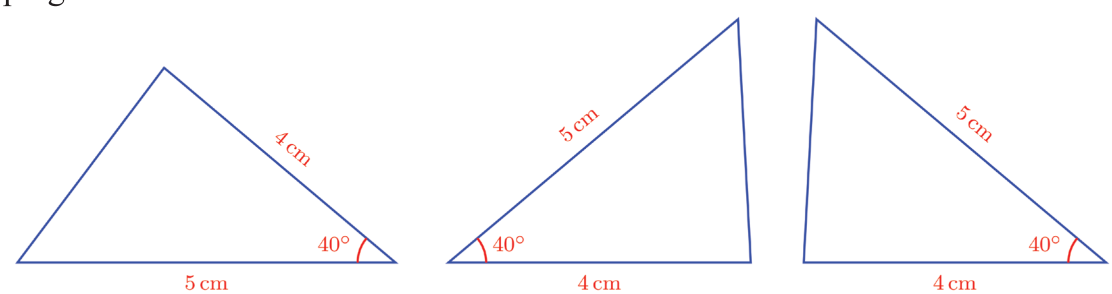
Now we have to place another piece of eerkkil over the two sides of the angle, to make a triangle. We can put it in different positions, can't we ?



Let's mark a spot on the top side of the angle. What if we insist that the second eerkkil should pass through this ?



Now let's mark spots on both the top and bottom sides and want the second piece to pass through both these. How many triangles can we make ? Once we fix one angle and the lengths of its two sides, a triangle is determined, isn't it ?



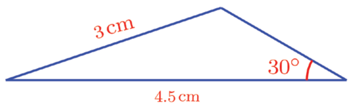
But this does not change the length of the third side or the other two angles.

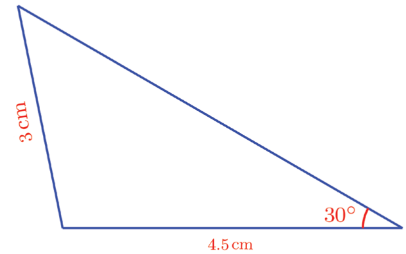
What we have seen in this example can be stated as a general principle:

If in two triangles, the lengths of two sides and the angle between them are the same, then the length of the third side and the remaining two angles are also the same

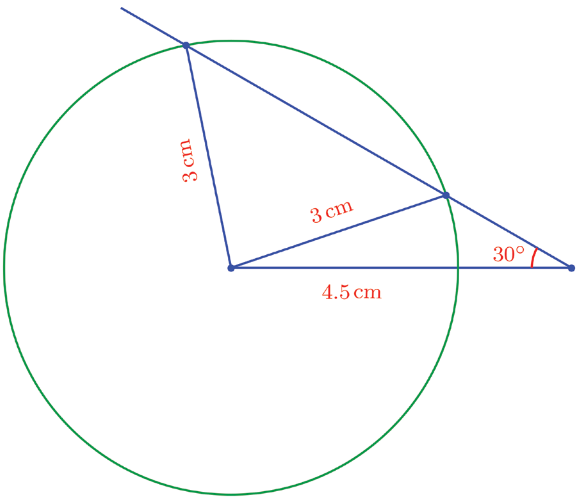
Would this be true if instead of the angle between the sides, some other angles are the same ?

See these pictures :





We have drawn such triangles in class 7



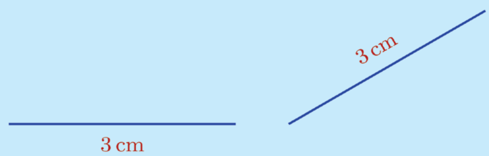
In these two triangles, the lengths of two sides and one of the angles are the same; but the length of the third side and the other two angles are not the same.

In two triangles, even if the lengths of two sides and one angle are the same, the third side and the other two angles may not be the same.

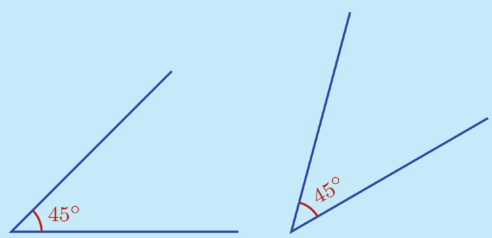
Equality

There are all kinds of geometric figures like lines, angles, rectangles, triangles and so on.

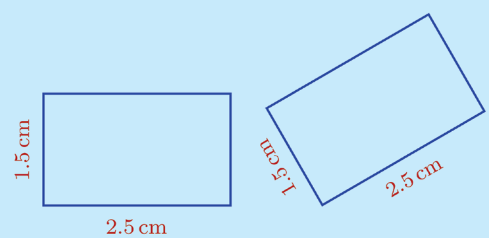
Two lines of the same length, however they are drawn are said to be equal:



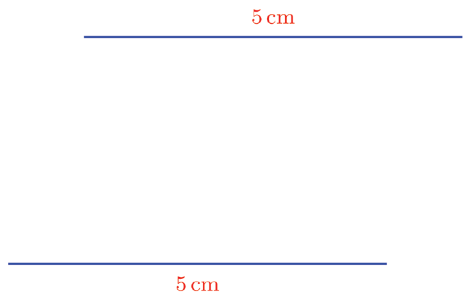
Similarly all angles of the same size are said to be equal:



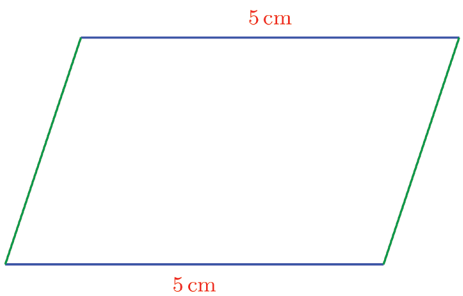
Rectangles with sides of the same length can also be said to be equal:



Now let's look at a problem. Draw two parallel lines of the same length:

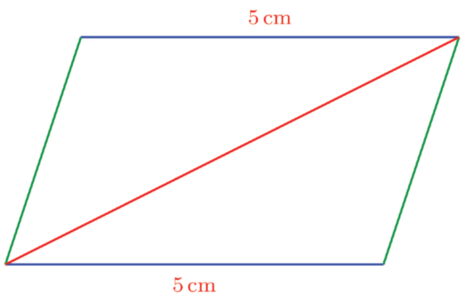


And join the ends as shown below:



What can we say about the green lines on the left and right ?

Don't they seem to be of the same length ? How do we make sure ?

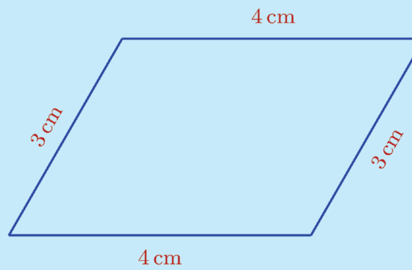


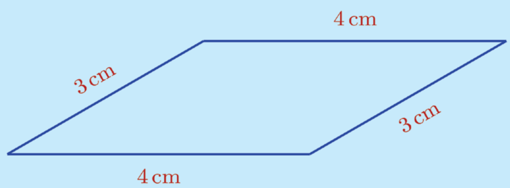
In both triangles, the blue sides have the same length.

And the red line is a side of each triangle.

Geometric equality

See these parallelograms :





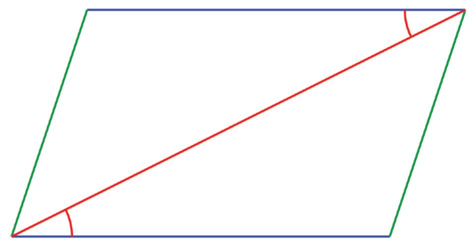
The lengths of sides of both are the same; but we can't say they are equal, can we ?

This is what Euclid says about equality of geometric figures:

> *Things which coincide with one another equal one another*

The lines, angles and rectangles shown in the previous page do coincide when rotated, don't they ?

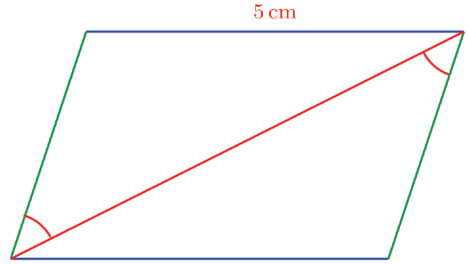
What about the angles between them ?



These are the small angles made by the red line with the blue parallel lines and so are equal.

Thus in the top and bottom triangles, the length of two sides and the angle between them are equal.

So, the green lines, which are the third sides of these triangles, are also equal; not only that, the angles marked in the picture below are also equal:



That is, the small angles made by the red line with the green lines are equal.

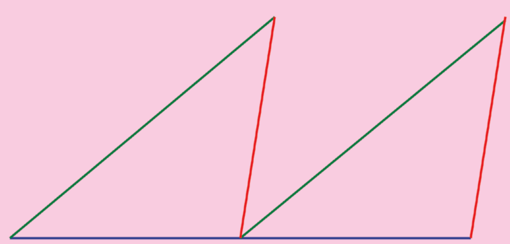
So, the green lines are also parallel.

Thus the quadrilateral is a parallelogram.

This fact can be stated as a general principle

If two sides of a quadrilateral are equal and parallel , then the quadrilateral is a parallelogram

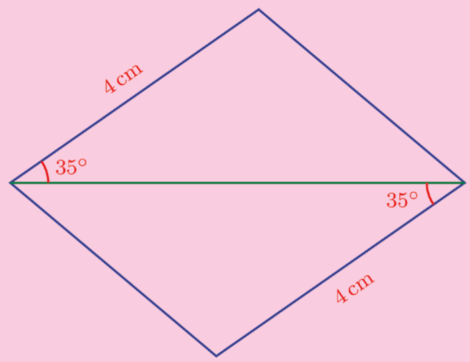
(1) The green lines in the picture below are parallel and of the same length; one is drawn from the end of the horizontal blue line and the other is drawn from the midpoint of the blue line :



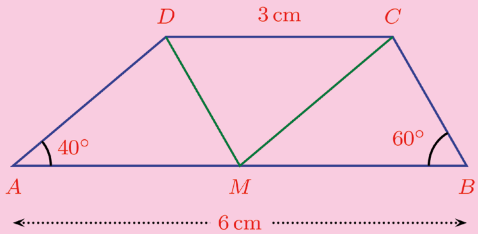
(i) Are the lengths of the red lines in the picture equal ? Why ?

(ii) Are the red lines parallel ?

(2) Is the quadrilateral below a parallelogram ? Why ?



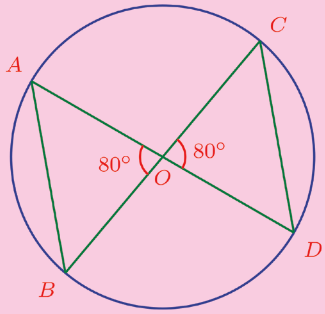
(3) In the figure below, the lines AB and CD are parallel. M is the midpoint of AB:



(i) Are the quadrilaterals *AMCD* and *MBCD* parallelograms ? Why ?

(ii) Calculate all the angles of the triangles *AMD*, *MBC*, *DCM*.

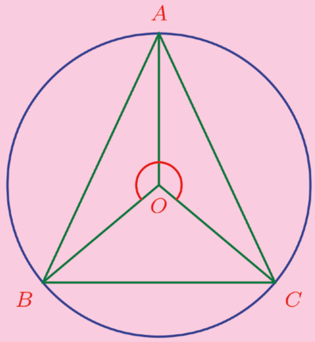
(4) In the picture, O is the centre of the circle and A, B, C, D are points on the circle:



Are the lines *AB* and *CD* equal ? Why ?

(5) In the picture, *O* is the centre of the circle and *A*, *B*, *C* are points on the circle. ∠AOB and ∠AOC are equal:

Are the lines AB and AC equal ? Why ?



Let's summarize what we have seen so far.

A triangle has six measures, the lengths of three sides and the sizes of the three angles. If all these six measures are the same in two triangles, we can so place them one over the other that they coincide.

In other words, they are **equal triangles.**

To check whether two triangles are equal, should we compare all these six measures? What we have seen is that we need check only three measures, which are specially chosen:

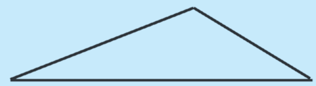
If in two triangles, any of the sets of three measures listed below are the same , then the other three measures are also the same; that is, they are equal triangles

- **Three sides**
- **Two sides and the angle between them**
- **One side and the angles at its ends**

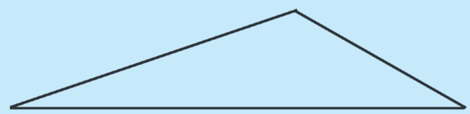
Incorrect match

A triangle has three sides and three angles and thus six measures in all. We have seen that if in two triangles certain triples of these measures (three sides, two sides and the included angle, one side and the angles on them) are equal, then these triangles are equal; that is, the remaining measures are also equal.

Now take a large sheet of paper and draw a triangle of sides 4, 6 and 9 centimetres.



And then a triangle of sides 6, 9 and 13.5 centimetres.

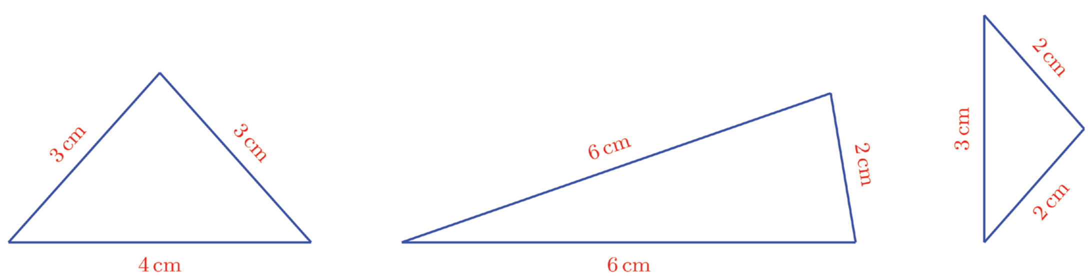


Measure their angles. The angles of the two triangles are equal, aren't they ? (You can also check this by cutting out the triangles and placing each angle of one triangle over the other).

Thus in these two triangles five of the six measures (three angles and two sides) are equal; but they are evidently not equal.

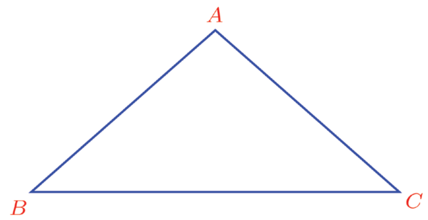
Isosceles triangles

See these triangles:



In all these triangles, two of the sides are equal . Such triangles are called **isosceles triangles.**

See this isosceles triangle:



The left and right sides are of the same length.

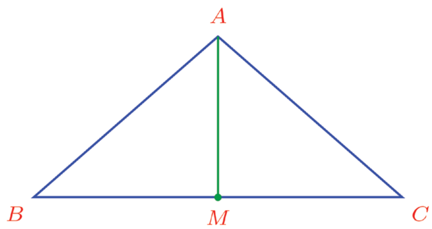
Look at the angles on the left and right ? Notice anything ?

Cut out such a triangle from a sheet of paper and fold it along the middle so that the equal sides coincide.

The angles on the left and right also coincide, don't they ?

Why are the angles equal ?

Let's draw the line of fold in the picture; that is join the top vertex with the midpoint of the bottom side:



Look at the sides of the small triangles ABM and ACM on the left and right.

- $AB = AC$ as mentioned at the beginning
- $BM = CM$ since M is the midpoint of BC
- AM is a side of both these triangles

Thus the lengths of the sides of the triangle ABM and ACM are the same.

So, the angles opposite equal sides must also be the same.

Thus $\angle B = \angle C$

These are the angles opposite the equal sides of the first triangle ABC, right ?

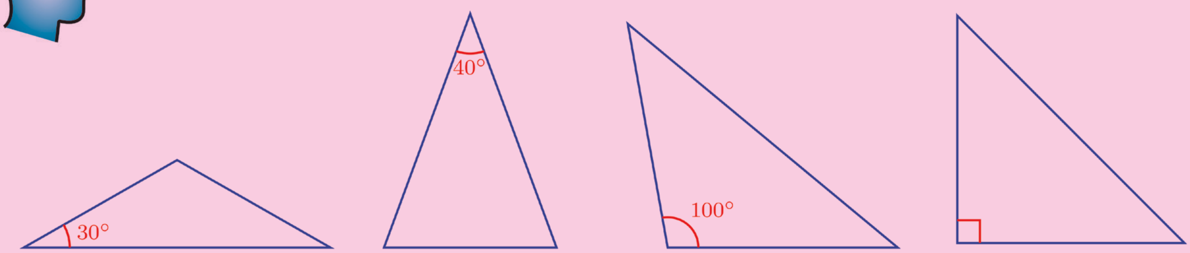
This we can state as a general result :

If two sides of a triangle are equal , then their opposite angles are also equal

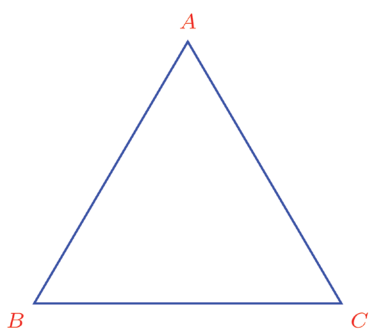
In GeoGebra, make a slider **a** with Min : 3 and Max : 15. Draw line AB of length 6. Draw circles with centres at A and B and with radius **a** and mark a point C where these intersect. Draw triangle ABC. Now the circles can be hidden. As we change the value of **a** using the slider, we get different triangles, with two sides equal. What about the angles ?

What are the angles when **a** is 6 ?

Find the other angles in each of the isosceles triangles below ?



Now a question: what if all three sides of a triangle are equal ?



In GeoGebra select Slider and the option Angle in its dialogue window. The name would be **a**. Set Min : 0 and Max : 90 °

Draw the line *AB* of length 6 and draw lines at its ends such that ∠A = ∠B = **a**. Mark their point of intersection as *C* and draw the triangle ABC.

See how the lengths of sides change as **a** changes. What is the specialty of the triangle got when **a** is 60° ? And when **a** is 45° ?

In the triangle *ABC* above, all three sides are of the same length.

Since the sides *AB* and *AC* are equal, the angles at *B* and *C* are equal, by the statement above.

Since the sides *BA* and *BC* are equal, the angles at *A* and *C* are equal.

So the three angles are all equal, right ?

Since the sum of the three angles is 180°, we can also compute each angle as $180° \div 3 = 60°$

A triangle with all three sides equal is called an **equilateral triangle**. So, we have a general result:

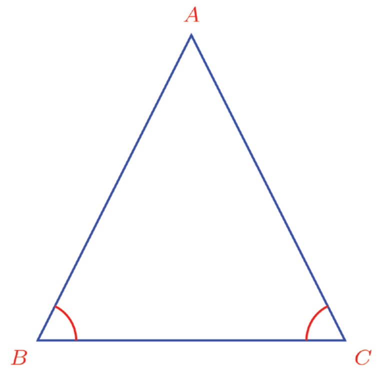
In any equilateral triangle, each angle is 60°

Now another question:

If we put the first statement the other way round, would it also be true ?

That is, if two angles of a triangle are equal, would the sides opposite them also be equal ?

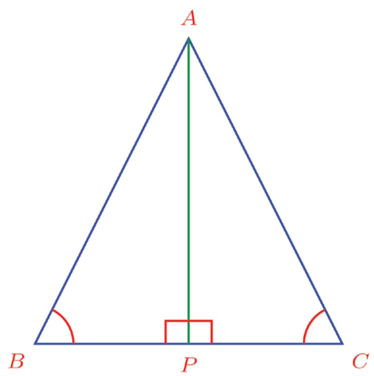
See this picture :



In this triangle *ABC*, the angles at *B* and *C* are equal. The question is whether *AB* and *AC* have equal length.

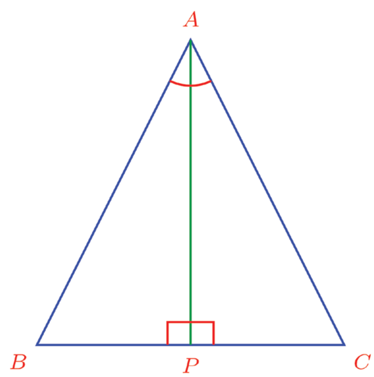
As before , let's split the triangle into two smaller ones.

Here it is more convenient to draw the perpendicular from the top vertex to the bottom side, instead of the line joining the top vertex and the midpoint of the side.



Now in the triangles *ABP* and *ACP* on the left and right, the angles at the vertices *B* and *C* are equal. In both triangles, the angles at *P* are equal each being a right angle.

So, the third angles of the triangles must also be equal:



The line AP is a side of each of the triangles.

Thus in both triangles, one side and the angles at its ends are the same. So, the sides AB and AC, which are opposite to the right angle in each, are equal.

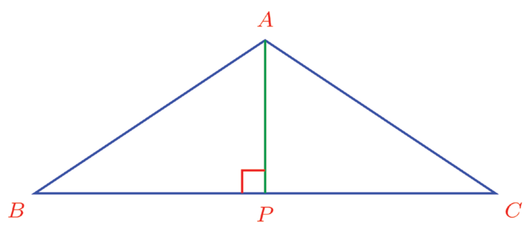
These are the opposite sides of the equal angles in the triangle ABC.

So, our result is true the other way round also.

If two angles of a triangle are equal, then their opposite sides are also equal

We can see another thing here:

In the picture below, an isosceles triangle, and the perpendicular from the vertex joining the equal sides to the opposite side, are drawn:



Since AB and AC are equal, $\angle B$ and $\angle C$ are also equal. These are angles in the right triangles ABP and ACP. Thus two of the angles of each triangle are equal. So, the third angles must also be equal. That is

$$\angle BAP = \angle CAP$$

What does this mean ?

The line AP splits $\angle BAC$ into two equal parts.

Again, in both triangles the side AP is the same and the angles at its ends are also the same.

So, their third sides must also be equal.

$$BP = CP$$

That is, the perpendicular AP splits the side BC into two equal parts

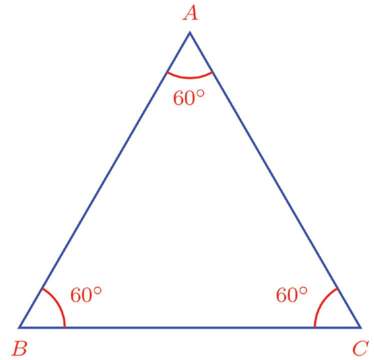
We can state these as a general result :

In any isosceles triangle, the perpendicular from the vertex joining the equal sides to its opposite side splits this side and the angle at this vertex into equal parts

Now another question:

We have seen that in any equilateral triangle, each angle is 60°

On the other hand, if all angles of a triangle are equal to 60°, would it be equilateral ?



We have seen that if two angles of a triangle are equal, then their opposite sides are equal.

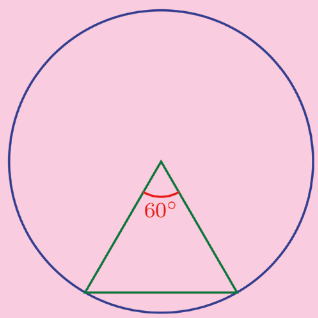
Since $\angle B = \angle C$ in the picture, we have $AC = AB$

Again, since $\angle A = \angle C$, we also get $BC = AB$

Thus $AB = BC = AC$ and so the triangle is equilateral.

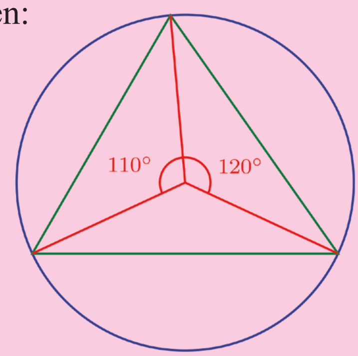
(1) One angle of an isosceles triangle is 120°. What are the other two angles ?

(2) The picture shows a triangle drawn by joining the centre of a circle and two points on the circle:



Calculate the other two angles of the triangle .

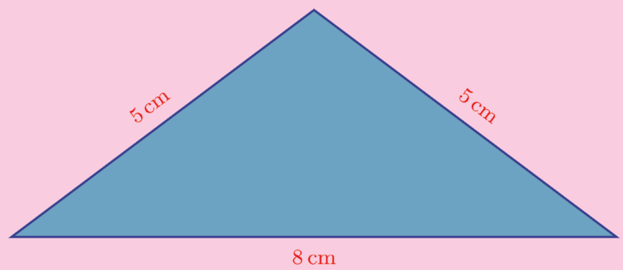
(3) The picture shows the triangle drawn by joining three points on a circle. Two of the angles formed by joining these points to the centre of the circle are also given:

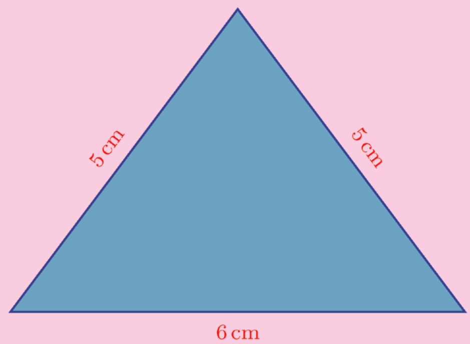


(i) Calculate the third angle at the centre.

(ii) Calculate all the angles of the large (green) triangle in the circle.

(4) Calculate the areas of each of the triangles below :





(5) Given that one angle of an isosceles triangle is 70°. What can we say about the other angles ?

(6) How many non - equal isosceles triangles can be drawn with one angle 70° and one side 8 centimetres ?

3

Square Identities

Squares of sums

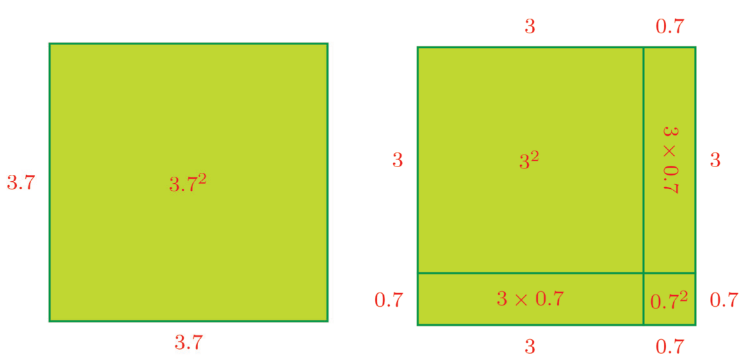
We have seen a method to calculate the squares of many numbers in the section **Squares**. For example,

	30	6
30	900	180
6	180	36

$$36^2 = 30^2 + (2 \times 30 \times 6) + 6^2$$
$$= 900 + 360 + 36$$
$$= 1296$$

And how did we compute 3.7^2 ?

By considering the area of a square of side 3.7, right? (The section **Decimal squares** of the lesson **Squares**).



$$3.7^2 = 3^2 + (2 \times 3 \times 0.7) + (0.7)^2$$
$$= 9 + 4.2 + 0.49$$
$$= 13.69$$

In the first problem, we wrote 36 as the sum 30 + 6 to calculate its square; and in the second, we split 3.7 as the sum 3 + 0.7

How did we then calculate the square ?

In the first problem, what all numbers did we add to get 36^2 ?

$$30^2 = \text{Square of } 30$$
$$6^2 = \text{Square of } 6$$
$$2 \times 30 \times 6 = \text{Twice the product of 30 and 6}$$

And in the second problem, what all numbers did we add to get 3.7^2 ?

$$3^2 = \text{Square of } 3$$
$$0.7^2 = \text{Square of } 0.7$$
$$2 \times 3 \times 0.7 = \text{Twice the product of 3 and 0.7}$$

We can compute the square of any sum like this. How do we state this as a general rule?

The square of the sum of two numbers is equal to the sum of the squares of these numbers and twice their product

We have seen in class 7 such instances of writing relations between operations on numbers in shorthand, using algebra (The section **Numbers and algebra** of the lesson **Algebra**).

How do we write the above result on squares using algebra ?

If the numbers are written x and y, then their sum is $x + y$.

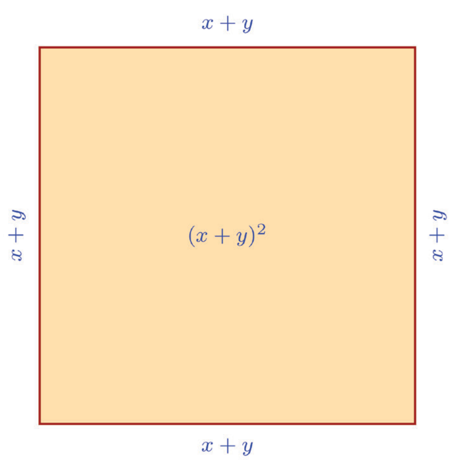
What about its square?

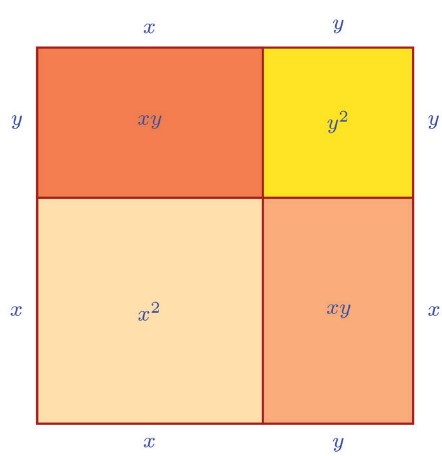
We write it as $(x + y)^2$

So, what's the algebraic form of our result ?

$(x + y)^2 = x^2 + y^2 + 2xy$ for all numbers x and y

We can see it geometrically as a result on areas:





In the equation $(x + y)^2 = x^2 + y^2 + 2xy$, we can take any two numbers as x and y.

Suppose we take y as 1. We get

$$(x + 1)^2 = x^2 + 1 + 2x$$

In algebraic expressions, we usually write letters first. So, we write the above equation as

$$(x + 1)^2 = x^2 + 2x + 1$$

Here, we can take any number as x.

Thus we have a special case of the general result above:

$$(x + 1)^2 = x^2 + 2x + 1 \text{ for all numbers } x$$

Thus to get the square of one more than a number, we need only add the square of the number, twice the number and one.

What do we get when we take x as 1, 2, 3, ... in this ?

$$2^2 = 1^2 + (2 \times 1) + 1 = 1 + 2 + 1 = 4$$

$$3^2 = 2^2 + (2 \times 2) + 1 = 4 + 4 + 1 = 9$$

$$4^2 = 3^2 + (2 \times 3) + 1 = 9 + 6 + 1 = 16$$

....

$$11^2 = 10^2 + (2 \times 10) + 1 = 100 + 20 + 1 = 121$$

$$12^2 = 11^2 + (2 \times 11) + 1 = 121 + 22 + 1 = 144$$

....

(We have seen this in the section **Perfect squares** of the lesson **Squares**).

Now let's take y as $\frac{1}{2}$ in the identity $(x + y)^2 = x^2 + y^2 + 2xy$:

$$\left(x + \frac{1}{2}\right)^2 = x^2 + \left(\frac{1}{2}\right)^2 + \left(2 \times x \times \frac{1}{2}\right)$$

$$= x^2 + \frac{1}{4} + x$$

$$= x^2 + x + \frac{1}{4}$$

Thus we get another special case:.

$$\left(x + \frac{1}{2}\right)^2 = x^2 + x + \frac{1}{4} \text{ for all numbers } x$$

If we take x as the natural numbers, we get

$$\left(1\tfrac{1}{2}\right)^2 = 1^2 + 1 + \frac{1}{4} = 1 + 1 + \frac{1}{4} = 2\tfrac{1}{4}$$

$$\left(2\tfrac{1}{2}\right)^2 = 2^2 + 2 + \frac{1}{4} = 4 + 2 + \frac{1}{4} = 6\tfrac{1}{4}$$

$$\left(3\tfrac{1}{2}\right)^2 = 3^2 + 3 + \frac{1}{4} = 9 + 3 + \frac{1}{4} = 12\tfrac{1}{4}$$

The decimal form of this we have seen in the lesson **Squares** (the section **Decimal squares**).

Now look at this equation

For any non-zero number x, we have

$$\left(x + \frac{1}{x}\right)^2 = x^2 + \frac{1}{x^2} + 2 \times x \times \frac{1}{x}$$

$$= x^2 + \frac{1}{x^2} + 2$$

How about taking x as 2, 3, 4, ... in this ?

$$\left(2\tfrac{1}{2}\right)^2 = 2^2 + \frac{1}{2^2} + 2 = 6\tfrac{1}{4}$$

$$\left(3\tfrac{1}{3}\right)^2 = 3^2 + \frac{1}{3^2} + 2 = 11\tfrac{1}{9}$$

$$\left(4\tfrac{1}{4}\right)^2 = 4^2 + \frac{1}{4^2} + 2 = 18\tfrac{1}{16}$$

....

(1) Calculate the squares below in head:

(i) 15^2 (ii) 25^2 (iii) 33^2 (iv) $\left(5\tfrac{1}{2}\right)^2$ (v) $\left(10\tfrac{1}{2}\right)^2$ (vi) $\left(25\tfrac{1}{2}\right)^2$ (vii) $\left(5\tfrac{1}{5}\right)^2$

(2) In the general identity on the square of a sum of two numbers, what special case do we get by taking one of the numbers as 2? Do the calculations below in head, using this:

(i) What is 22^2 ?

(ii) What is 52^2 ?

(iii) $25^2 = 625$; What is 27^2 ?

(3) $43^2 = 1849$

(i) What is 44^2 ? (ii) What is 46^2 ?

The identity about the square of a sum of two numbers can also be used to understand the peculiarity of squares of some special numbers

For example, see these squares:

15^2	25^2	35^2
$= (10+5)^2$	$= (20+5)^2$	$= (30+5)^2$
$= 10^2 + (2 \times 10 \times 5) + 5^2$	$= 20^2 + (2 \times 20 \times 5) + 5^2$	$= 30^2 + (2 \times 30 \times 5) + 5^2$
$= 100 + 100 + 25$	$= 400 + 200 + 25$	$= 900 + 300 + 25$
$= 225$	$= 625$	$= 1225$

Does the square of any number with last digit (the digit in the one's place) 5 have 25 as the last two digits combined ?

Let's check using algebra:

The last digit of any number is the remainder on division by 10, isn't it?

(The section **Remainders** of the lesson **Number Relations** in the class 5 textbook).

So, a number ending in 5, divided by 10 leaves remainder 5.

If the quotient of this division is taken as n, the general form of such a number is $10n + 5$ (Recall the section **Multiple and remainder** of the lesson **Algebra** in the class 7 textbook).

What is the square of a number of this form?

$$(10n+5)^2 = (10n)^2 + (2 \times 10n \times 5) + 5^2$$
$$= 100n^2 + 100n + 25$$

In this, we can write $100n^2 + 100n$ like this:

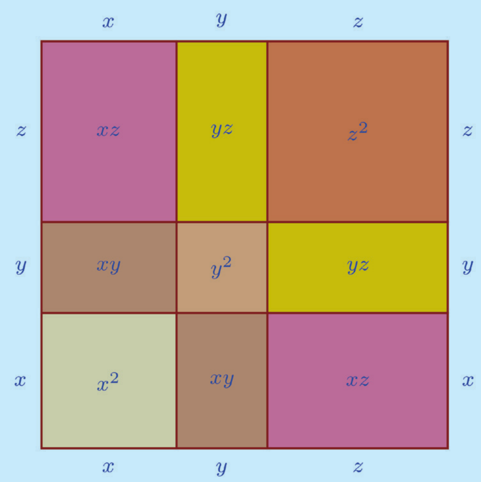
$$100n^2 + 100n = 100(n^2 + n)$$

So,

$$(10n+5)^2 = 100(n^2+n) + 25$$

Three numbers

We calculated the square of a sum of two numbers geometrically by dividing a square into four parts and adding. We can calculate the square of a sum of three numbers also like this:



From this, we can see that

$$(x+y+z)^2 = x^2 + y^2 + z^2 + 2(xy + yz + zx)$$

This means, the square is got by multiplying the number $n^2 + n$ by 100 and then adding 25.

Any number multiplied by 100 has the last two digits zero; so when 25 is added, the last two digits together make 25.

For example,

$$\begin{aligned} 65^2 &= ((6^2 + 6) \times 100) + 25 \\ &= (42 \times 100) + 25 \\ &= 4200 + 25 \\ &= 4225 \end{aligned}$$

We can also calculate the number formed by the other digits in such a square.

For that we write the $n^2 + n$ in the equation $(10n + 5)^2 = 100(n^2 + n) + 25$ as

$$n^2 + n = n(n + 1)$$

so that we can write

$$(10n + 5)^2 = (100 \times n(n + 1)) + 25$$

From this, we can see that the number obtained by removing the last 25 from the square is $n(n + 1)$

Then n here is the number got by removing the last digit 5 from the original number. So, the number got by removing the 25 from the square can be described like this: remove the 5 from the number to be squared and multiply the number got by the next natural number

For example,

$$\begin{aligned} 75^2 &= (100 \times 7 \times 8) + 25 \\ &= 5600 + 25 \\ &= 5625 \end{aligned}$$

What about 195^2 ?

First compute 19×20:

$$19 \times 20 = 380$$

Now affix 25 at the end of this number to get the square of 195:

$$195^2 = 38025$$

Let's look at another fact.

Any number divided by 3 leaves remainder 0, 1 or 2.

If the remainder is 0, then the number is a multiple of 3. So, non-multiples of 3 leave remainder 1 or 2 on division by 3.

Repeating 76

See these squares:

$$76^2 = 5776$$

$$176^2 = 30976$$

$$276^2 = 76176$$

Calculate the square of some other numbers ending in 76. Notice anything peculiar?

Why is this so?

We can write any number ending in 76 as $100n + 76$; and

$$(100n + 76)^2 = 10000n^2 + 15200n + 5776.$$

Now whatever be the natural number n, the last two digits of $10000n^2 + 15200n$ are zeros; and adding 5776 to it makes the last two digits 76.

Does any two-digit number other than 76 have this property ?

What if we divide the squares of such numbers by 3?

Number	Square	Remainder on division by 3
1	1	1
2	4	1
4	16	1
5	25	1

Does the square of any such number leave remainder 1 on division by 3 ? Let's check using algebra:

All numbers which leave a remainder 1 on division by 3 can be written in the general form

$$3n + 1, \ (n = 0, 1, 2, 3, ...)$$

(The section **Multiple and remainder** of the lesson **Algebra** in the class 7 textbook).

What about their squares?

$$(3n + 1)^2 = (3n)^2 + (2 \times 3n \times 1) + 1^2$$

$$= 9n^2 + 6n + 1$$

In this, we can write $9n^2 + 6n$ as a multiple of 3:

$$9n^2 + 6n = 3(3n^2 + 2n)$$

so that

$$(3n + 1)^2 = 3(3n^2 + 2n) + 1$$

This means any such square is 1 added to a multiple of 3; in other words, it leaves remainder 1 on division by 3.

What about numbers which leave remainder 2 on division by 3?

Computer help

To make a table as above, we can write a program in Python:

```
for n in range(1,11):
    if n%3! = 0:
        print (n, n**2, (n**2)%3)
```

The first line is an instruction to take n as the natural numbers from 1 to 10.

In the second line, n%3 means the remainder on dividing n by 3; and ! = 0 means not equal to 0. So, the entire line if n%3!=0 means if the remainder on dividing n by 3 is not 0.

The last line instructs what to do with numbers satisfying this condition: write out such numbers, their squares and the remainders on dividing these squares by 3.

Running the program gives this table:

```
1   1    1
2   4    1
4   16   1
5   25   1
7   49   1
8   64   1
10  100  1
```

Their general form is

$$3n + 2, \quad (n = 0, 1, 2, 3, \ldots)$$

and the square of any such number is

$$(3n + 2)^2 = 9n^2 + 12n + 4 = 3(3n^2 + 4n) + 4$$

What is the remainder on dividing such a number by 3 ?

We can write the 4 in the equation above as 3 + 1.

Then

$$\begin{aligned}(3n + 2)^2 &= 3(3n^2 + 4n) + 3 + 1 \\ &= 3(3n^2 + 4n + 1) + 1\end{aligned}$$

Again we get 1 added to a multiple of 3.

Thus even if get remainder 2 on dividing a number by 3, the square of the number leaves remainder 1 on division by 3.

So, what can we say in general ?

The square of any non-multiple of 3 leaves remainder 1 on division by 3.

The square of any multiple of 3 is again a multiple of 3, right ? (Why?)

So, we have this general result

Any perfect square leaves remainder 0 or 1 on division by 3.

This can also be stated like this:

Any number which leaves remainder 2 on division by 3 is not a perfect square.

(1) Calculate the squares of some odd numbers and check the following statement. Explain why they are true:

(i) The square of any odd number is odd

(ii) The square of any odd number leaves remainder 1 on division by 4

(iii) The square of any odd number leaves remainder 1 on division by 8

(2) Calculate the remainders on dividing the squares of some natural numbers by 4. Explain why the statements below are true:

(i) The square of any natural number divided by 4 leaves remainder 0 or 1.

(ii) Any natural number which leaves remainder 2 or 3 on division by 4 is not a perfect square.

(3) See this pattern:

$$3 = 2^2 - 1^2$$

$$5 = 3^2 - 2^2$$

$$7 = 4^2 - 3^2$$

Check whether some other odd numbers can also be written as the difference of two perfect squares. Explain why all odd numbers greater than 1 can be written like this.

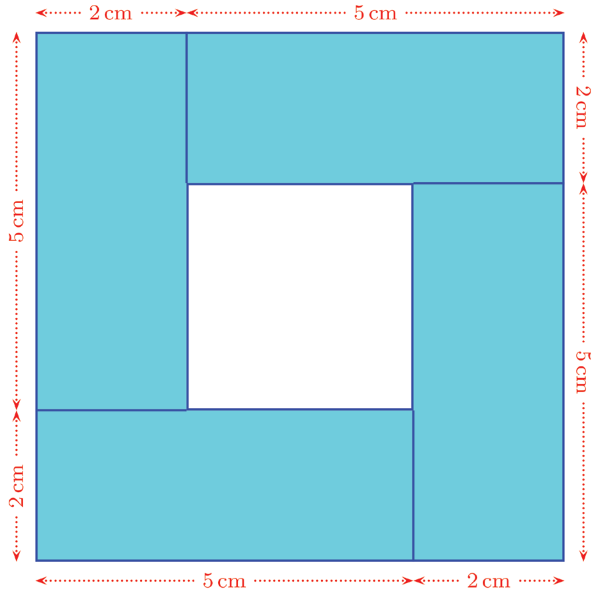
(Hint: Recall the general form of an odd number seen in class 7).

(4) Give reasons for the fact that the square of any number ending in 1, also end in 1. What about numbers ending in 5 ?

And numbers ending in 6 ?

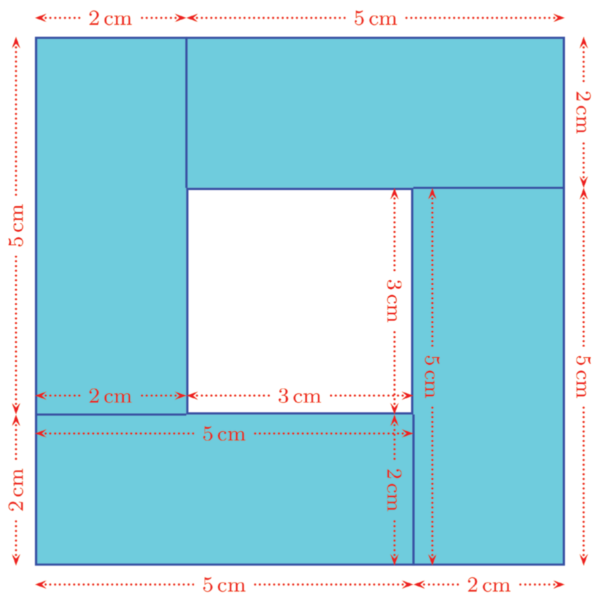
Squares of differences

See this picture:



A square is made with four identical rectangles. What is the area of the small rectangle within the large square ?

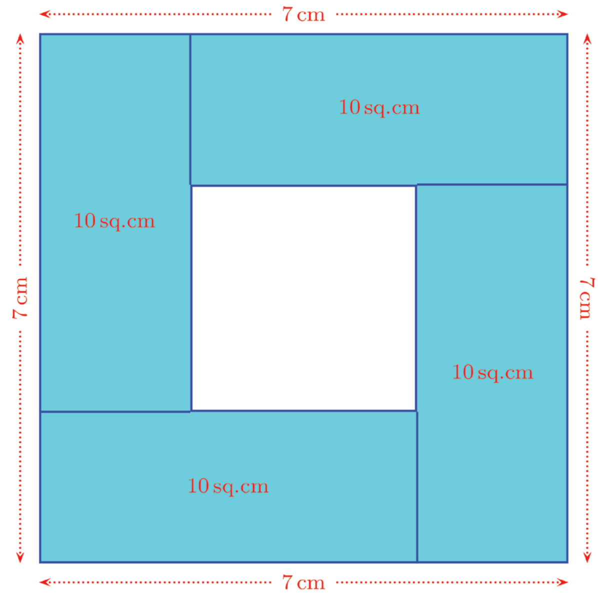
We can calculate the length of the sides of this small rectangle:



So it's in fact a square; and the length of a side is 3 centimetre. So the area is

$$3^2 = 9 \text{ sq.cm}$$

This can be calculated in a different way: Subtract from the area of the large square, the total area of the four rectangles:



So, the area is

$$7^2 - (4 \times 10) = 49 - 40 = 9 \text{ sq.cm}$$

How did we calculate the lengths of the sides of the small square ?

$$5 - 2 = 3$$

And the lengths of the sides of the large square ?

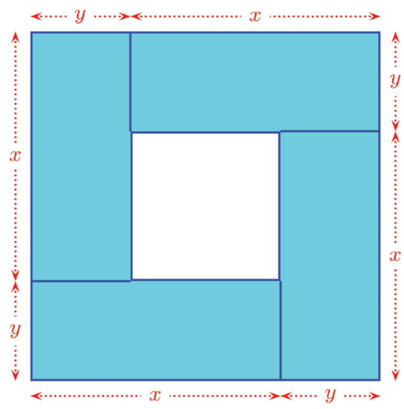
$$5 + 2 = 7$$

So, considering these numbers alone, what does the above computations give ?

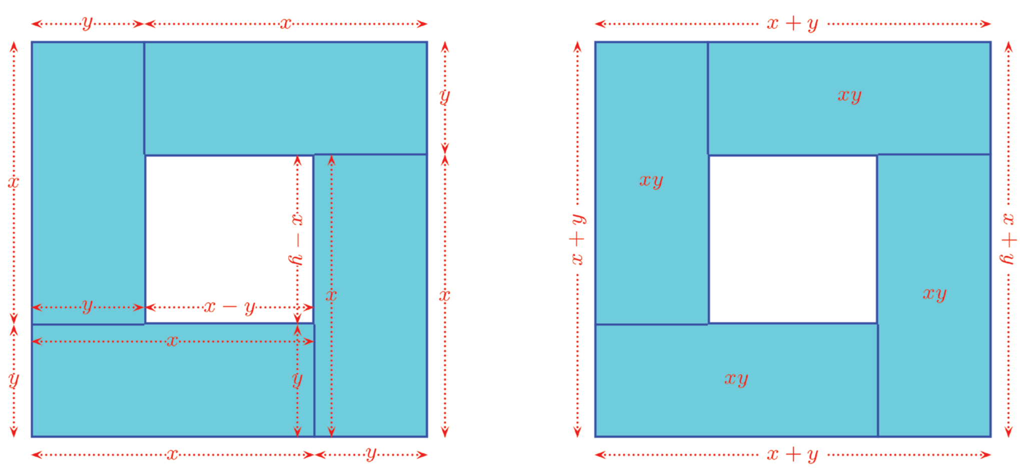
$$(5 - 2)^2 = (5 + 2)^2 - (4 \times 5 \times 2)$$

Does this hold for numbers other than 5 and 2 ?

Let's take the lengths of the sides of the four identical rectangles as x and y:



And let's compute the area of the small square within, in two different ways:



From the picture on the left,

$$\text{area of the small square is } (x-y)^2$$

From the picture on the right,

$$\text{area of the small square is } (x+y)^2 - 4xy$$

Thus we have

$$(x-y)^2 = (x+y)^2 - 4xy$$

In this, we can write $(x + y)^2$ like this :

$$(x + y)^2 = x^2 + y^2 + 2xy$$

using this in the above equation, we get

$$(x - y)^2 = (x^2 + y^2 + 2xy) - 4xy$$

The $4xy$ subtracted on the right side of the equation is $2xy + 2xy$, right ? So , to subtract $4xy$, we can first subtract a $2xy$ and then a $2xy$ again (Recall the section, **One by one and altogether** of the lesson, **Shorthand Math** in the class 7 textbook).

$$\begin{aligned}(x - y)^2 &= (x^2 + y^2 + 2xy) - 4xy \\ &= x^2 + y^2 + 2xy - 2xy - 2\mathrm{xy} \\ &= x^2 + y^2 - 2xy\end{aligned}$$

In this, x and y can be any two numbers. Thus

$(x - y)^2 = x^2 + y^2 - 2xy$ for all numbers x and y

How do we say this in ordinary language ?

The square of the difference of two numbers is twice their product subtracted from the sum of their squares

We can use this to quickly compute the square of some numbers

For example, to compute 99^2, instead of writing it as $(90 + 9)^2$ and computing it as the square of a sum , we can write it as $(100 - 1)^2$ and compute this as the square of a difference:

$$\begin{aligned}99^2 &= (100 - 1)^2 \\ &= 100^2 - (2 \times 100 \times 1) + 1^2 \\ &= 10000 - 200 + 1 \\ &= 9801\end{aligned}$$

Similarly

$$\begin{aligned} 48^2 &= (50-2)^2 \\ &= 50^2 - (2 \times 50 \times 2) + 2^2 \\ &= 2500 - 200 + 4 \\ &= 2304 \end{aligned}$$

(1) Calculate the squares below in head:

(i) 29^2 (ii) 38^2 (iii) 999^2 (iv) $\left(9\frac{1}{2}\right)^2$ (v) $(9.7)^2$

(2) See these computations:

$$3^2 - (2 \times 3) = 3 = 2^2 - 1$$

$$4^2 - (2 \times 4) = 8 = 3^2 - 1$$

$$5^2 - (2 \times 5) = 15 = 4^2 - 1$$

Explain the general principle of these using algebra.

From the equations about the squares of sums and differences, we can form other equations.

For example, adding the two equations

$$(x+y)^2 = x^2 + y^2 + 2xy$$
$$(x-y)^2 = x^2 + y^2 - 2xy$$

we get

$$(x+y)^2 + (x-y)^2 = 2(x^2+y^2)$$

Writing this in reverse,

$$2(x^2+y^2) = (x+y)^2 + (x-y)^2$$

Taking x and y as different numbers in this, we get relations like

$$\begin{aligned} 2(2^2+1^2) &= 10 = 3^2+1^2 \\ 2(3^2+2^2) &= 26 = 5^2+1^2 \\ 2(5^2+1^2) &= 52 = 6^2+4^2 \\ 2(6^2+4^2) &= 104 = 10^2+2^2 \end{aligned}$$

Thus whatever sum of two perfect squares we take, twice that can also be written as the sum of two other perfect squares.

Try writing 20, 40 and 80 as the sum of two perfect squares.

Let's look at another problem:

How did we get the identity about the square of a difference ?

First we got the equation

$$(x-y)^2 = (x+y)^2 - 4xy$$

What does this mean?

The number $4xy$ subtracted from the number $(x+y)^2$ gives the number $(x-y)^2$. So, the $(x-y)^2$ subtracted from $(x+y)^2$ must gives $4xy$, right ? That is

$$(x+y)^2 - (x-y)^2 = 4xy$$

Writing this in reverse

$$4xy = (x+y)^2 - (x-y)^2$$

If y is taken as 1 in this, we get

$$4x = (x+1)^2 - (x-1)^2$$

If we take x as 2, 3, 4 , ... in this, we get

$$8 = 3^2 - 1^2$$
$$12 = 4^2 - 2^2$$
$$16 = 5^2 - 3^2$$

....

Thus all multiples of 4, starting with 8 can be written as the difference of two perfect squares.

We have already seen that any odd number greater than 1 can be written as the difference of two perfect squares.

Let's see can all numbers be written as the difference of two squares?

Pythagorean triplet

Three natural numbers in which the square of a number is equal to the sum of the squares of other two, is called a Pythagorean triple. For example, since

$$3^2 + 4^2 = 5^2$$

3, 4, 5 form a Pythagorean triple. A clay tablet from Babylon, dated to be around 2000 BCE contains a list of several such triples.

There is a method to find all such triples. Take any two natural numbers m and n with $m > n$ and compute the numbers below:

$$a = m^2 - n^2$$
$$b = 2mn$$
$$c = m^2 + n^2$$

Then using the identity

$$(x-y)^2 + 4xy = (x+y)^2$$

We have

$$a^2 + b^2 = (m^2 - n^2)^2 + (2mn)^2$$
$$= (m^2 - n^2)^2 + 4m^2n^2$$
$$= (m^2 + n^2)^2$$
$$= c^2$$

This method was known to the Greek mathematicians of the third century *BCE*.

For that we write the equation $4xy = (x+y)^2 - (x-y)^2$ as

$$xy = \frac{1}{4}(x+y)^2 - \frac{1}{4}(x-y)^2$$

In this equation, if we write

$$\frac{1}{4}(x+y)^2 = \left(\frac{x+y}{2}\right)^2$$

and

$$\frac{1}{4}(x-y)^2 = \left(\frac{x-y}{2}\right)^2$$

then we have

$$xy = \left(\frac{x+y}{2}\right)^2 - \left(\frac{x-y}{2}\right)^2$$

What does this mean ?

The product of any two numbers (not necessarily natural numbers) is the difference of two squares (not necessarily perfect squares). More precisely

The product of any two numbers is the difference of the squares of half their sum and half their difference

We can write any number x as $x \times 1$. So , by the above result

$$x = \left(\frac{x+1}{2}\right)^2 - \left(\frac{x-1}{2}\right)^2$$

For example,

$$11 = \left(\frac{11+1}{2}\right)^2 - \left(\frac{11-1}{2}\right)^2 = 6^2 - 5^2$$

$$10 = \left(\frac{10+1}{2}\right)^2 - \left(\frac{10-1}{2}\right)^2 = \left(5\tfrac{1}{2}\right)^2 - \left(4\tfrac{1}{2}\right)^2$$

We can use the above idea to compute some products also.

For example, consider 135×65

Finding half the sum and difference of the numbers, we get

$$\frac{1}{2} \times (135 + 65) = 100$$

$$\frac{1}{2} \times (135 - 65) = 35$$

Then by the general result above, we have

$$135 \times 65 = 100^2 - 35^2$$
$$= 10000 - 1225$$
$$= 8775$$

As another example, We can compute 26.5×23.5 like this:

$$26.5 \times 23.5 = \left(\frac{26.5 + 23.5}{2}\right)^2 - \left(\frac{26.5 - 23.5}{2}\right)^2$$
$$= 25^2 - 1.5^2$$
$$= 625 - 2.25$$
$$= 622.75$$

(1) See these computations:

$$\left(\frac{1}{2}\right)^2 + \left(1\frac{1}{2}\right)^2 = 2\frac{1}{2} \qquad 2 = 2 \times 1^2$$
$$\left(1\frac{1}{2}\right)^2 + \left(2\frac{1}{2}\right)^2 = 8\frac{1}{2} \qquad 8 = 2 \times 2^2$$
$$\left(2\frac{1}{2}\right)^2 + \left(3\frac{1}{2}\right)^2 = 18\frac{1}{2} \qquad 18 = 2 \times 3^2$$

(i) Write the next two lines following this pattern.

(ii) Explain the general principle of these using algebra.

(2) Some natural numbers can be written as the difference of two perfect squares in two different ways. For example

$$24 = 7^2 - 5^2 = 5^2 - 1^2$$
$$32 = 9^2 - 7^2 = 6^2 - 2^2$$
$$40 = 11^2 - 9^2 = 7^2 - 3^2$$

Write the next few multiples of 8 as the difference of two perfect squares in two different ways. Explain algebraically how we can do this for all multiples of 8 starting with 24.

(3) See how some numbers are written as the difference of two perfect squares in two different ways:

$$15 = 8^2 - 7^2 = 4^2 - 1^2$$

$$21 = 11^2 - 10^2 = 5^2 - 2^2$$

$$35 = 18^2 - 17^2 = 6^2 - 1^2$$

(i) What are the different ways in which the numbers 15, 21 and 35 can be written as the product of two factors?

(ii) Find two more numbers of this type, which can be written as the difference of two perfect squares in two different ways.

(iii) What general result can we form from this?

(4) Compute the following products by writing them as the difference of two squares.

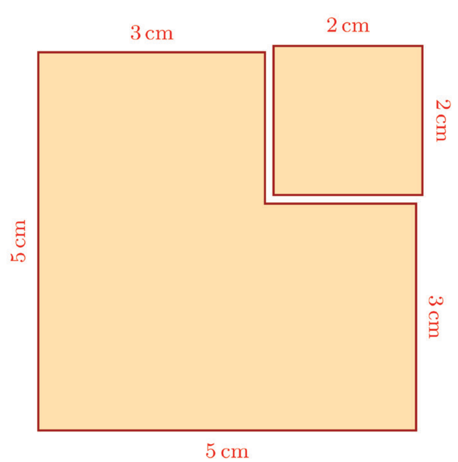
(i) 78×22 (ii) 301×299 (iii) $2\frac{1}{3} \times 1\frac{2}{3}$ (iv) 10.7×9.3

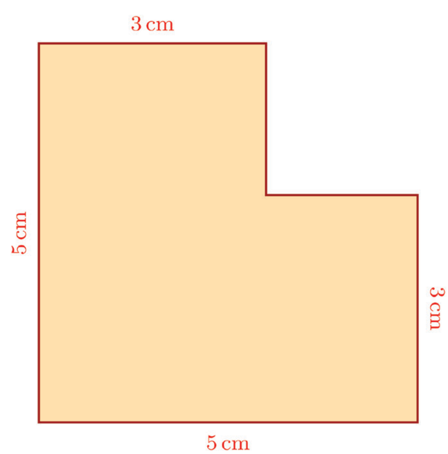
(5) Find the largest of each of the following pair of products, without actual multiplication:

(i)	75×25	76×24
(ii)	76×24	74×26
(iii)	10.6×9.4	10.4×9.6

Differences of squares

See these pictures:





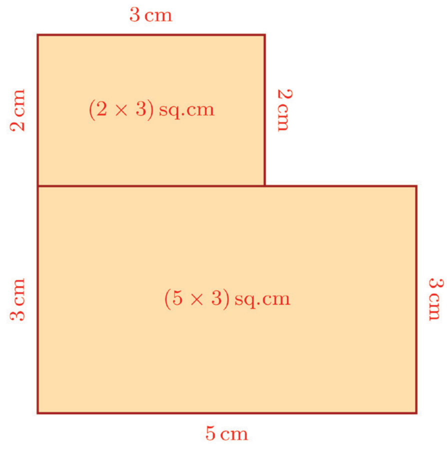
A small square is removed from a corner of a large square. What is the area of the remaining part?

Easy, isn't it?

Just subtract the area of the small square from the area of the large square:

$$5^2 - 2^2 = 25 - 4 = 21 \text{ sq.cm}$$

We can calculate this in another way. The remaining part can be split into two rectangles; just add their areas:



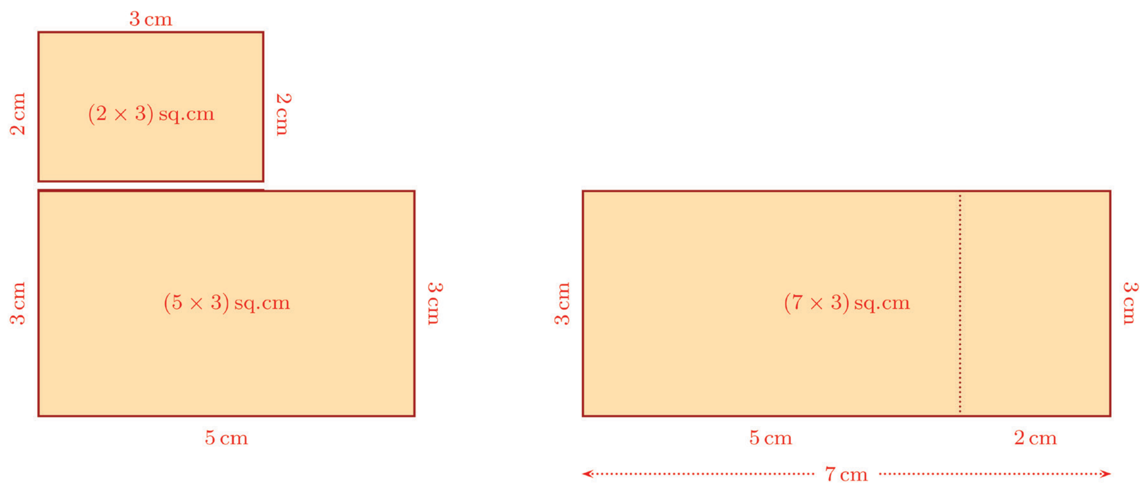
Area of remaining part is

$$(5 \times 3) + (2 \times 3) = 15 + 6 = 21 \text{ sq.cm}$$

This computation can also be done like this:

$$(5 \times 3) + (2 \times 3) = (5 + 2) \times 3 = 7 \times 3 = 21$$

Here we multiplied 7 and 3. It gives the area of a rectangle of sides 7 and 3, right ? So can we change the remaining part, a rectangle like this ?



We just cut out the small rectangle at the top and join it on the right of the large rectangle.

Writing this change in terms of numbers, we find

$$5^2 - 2^2 = 7 \times 3$$

where the 7 and 2 on the right side are got as

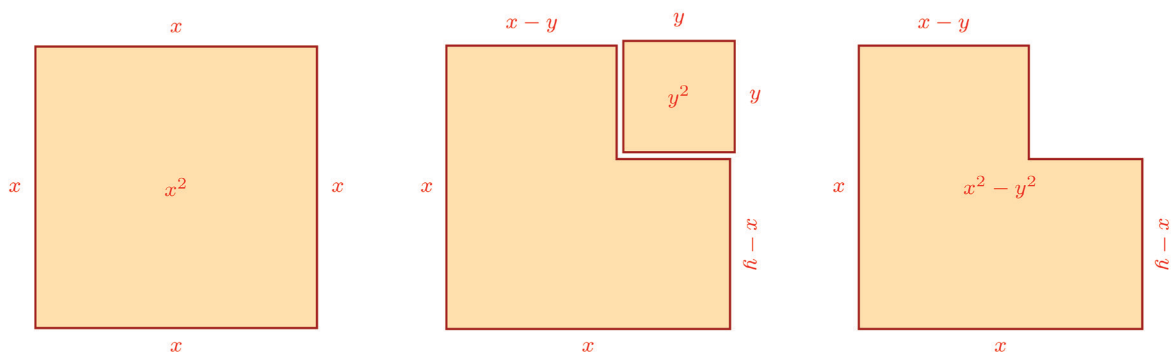
$$7 = 5 + 2$$
$$3 = 5 - 2$$

Thus we have

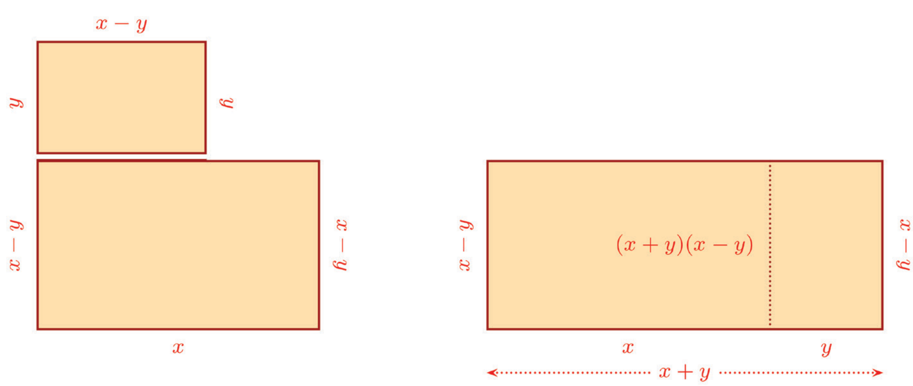
$$5^2 - 2^2 = (5 + 2) \times (5 - 2)$$

Can we write the difference of any two squares as the product of sum and difference, as above ?

Let's draw pictures, naming the numbers x and y:



Let's cut out a small rectangle from the top of the remaining part and join it on the right, as we did earlier:



So, now we can write this as a general result :

$x^2 - y^2 = (x + y)(x - y)$ **for all numbers** x **and** y

How do we say it in ordinary language ?

The difference of the squares of two numbers is the product of the sum and difference of the numbers

In the last section, we saw how we can write the product of two numbers as the difference of two squares. On the other hand, here we see how we can write the difference of two squares as a product.

For example,

$$168^2 - 162^2 = (168 + 162) \times (168 - 162)$$
$$= 330 \times 6$$
$$= 1980$$

We can do this for fractions also :

$$\left(5\tfrac{1}{2}\right)^2 - \left(4\tfrac{1}{2}\right)^2 = \left(5\tfrac{1}{2} + 4\tfrac{1}{2}\right) \times \left(5\tfrac{1}{2} - 4\tfrac{1}{2}\right)$$
$$= 10 \times 1 = 10$$

In the equation $x^2 - y^2 = (x + y)(x - y)$, if we take y as 1, then we get

$$x^2 - 1 = (x + 1)(x - 1)$$

This can also be written as $(x - 1)(x + 1)$.

Reading this in the other direction,

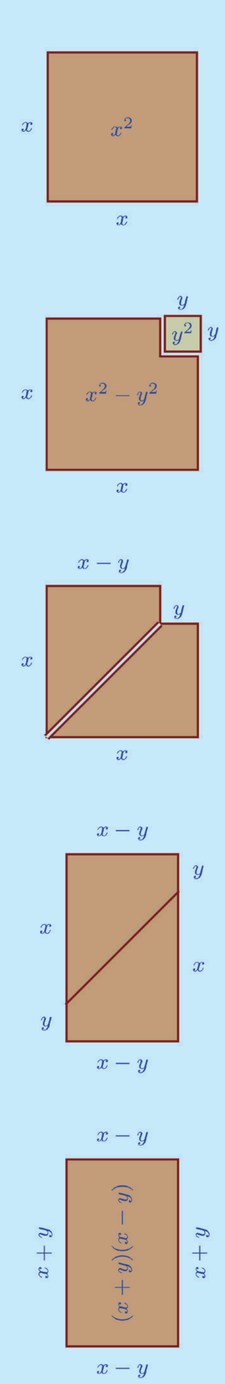
$$(x - 1)(x + 1) = x^2 - 1$$

Taking x as 2, 3, 4, ... in this we get

$$1 \times 3 = 2^2 - 1$$
$$2 \times 4 = 3^2 - 1$$
$$3 \times 5 = 4^2 - 1$$

....

Another method



How do we say this in ordinary language ?

For three consecutive natural numbers, the product of the first and the last is one less than the square of the middle number.

Recall what we saw in class 7 : for three consecutive natural numbers, the sum of the first and last is double the middle number (the section **Numbers and algebra** of the chapter **Algebra**).

(1) Do the computations below in head:

(i) $68^2 - 32^2$ (ii) $\left(3\frac{1}{2}\right)^2 - \left(2\frac{1}{2}\right)^2$ (iii) $3.6^2 - 1.4^2$

(2) Note the pattern in the computations below :

$$3^2 - 2^2 = 5 = 3 + 2$$
$$4^2 - 3^2 = 7 = 4 + 3$$
$$5^2 - 4^2 = 9 = 5 + 4$$

(i) Write the next two computations following these.

(ii) Write the general principle in these as an algebraic equation.

(iii) Write this general principle in ordinary language.

(3) In a calendar sheet, take a square containing nine numbers and mark the numbers on the left and right, top and bottom, of the middle number:

SUN	MON	TUE	WED	THU	FRI	SAT
			1	2	3	4
5	6	7	8	9	10	11
12	13	14	15	16	17	18
19	20	21	22	23	24	25
26	27	28	29	30	31	

Calculate the products of the left and right numbers and the top and bottom numbers, and also their difference

$$8 \times 10 = 80$$
$$80 - 32 = 48$$
$$2 \times 16 = 32$$

Do this for other squares of nine numbers. Explain using algebra, why the difference is 48 in all cases.

(*Hint : It is convenient to take the middle number as x*)

(4) As in the previous problem, mark a square of nine numbers in a calendar. Mark the four numbers in the corners:

SUN	MON	TUE	WED	THU	FRI	SAT
			1	2	3	4
5	6	7	8	9	10	11
12	13	14	15	16	17	18
19	20	21	22	23	24	25
26	27	28	29	30	31	

Calculate the products of the diagonal pairs of numbers and find their difference:

$$15 \times 3 = 45$$
$$45 - 17 = 28$$
$$1 \times 17 = 17$$

Do this for other squares of nine numbers. Explain using algebra, why the difference is 28 in all cases.

(5) A square has perimeter 20 centimetres. A rectangle has one side 2 centimetres longer and one side 2 centimetres shorter than a side of this square

(i) What is the perimeter of the rectangle ?

(ii) What are the areas of the square and the rectangle ?

(6) One side of a rectangle is longer than the side of a square and the other side is equally shorter than the side of the square.

(i) What can we say about the perimeters of the square and the rectangle ?

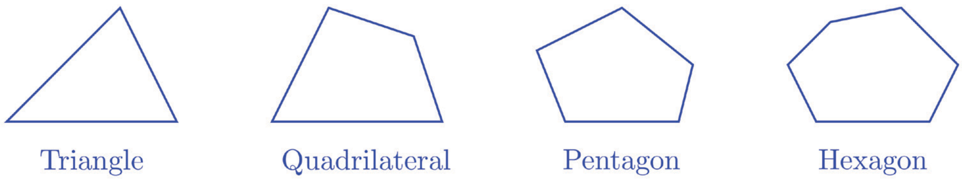
(ii) Which has the larger area , the square or the rectangle ?

We have seen that all odd numbers greater than 1 and certain even numbers can be written as the difference of two perfect squares. What kind of natural numbers cannot be written as the difference of two perfect squares ?

4 POLYGONS

Names

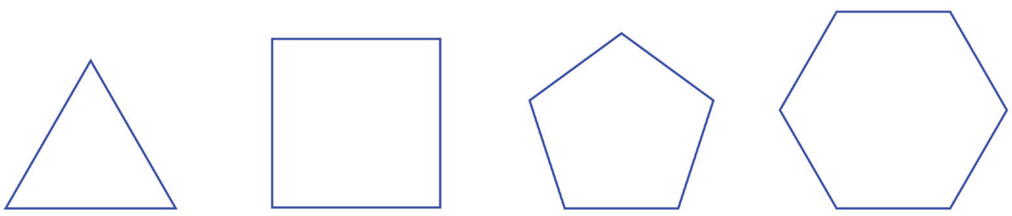
We have seen many geometrical figures made up of only straight lines:



A triangle has three vertices and three sides, while a quadrilateral has four vertices and four sides; five of each for pentagons, six for hexagons.

We can draw any number of such figures with more vertices and more sides. The general name for all such figures is **polygon**.

We can make the above pictures prettier like this:



Now all the polygons have sides of the same length and angles of the same size. Such a polygon is called a **regular polygon.**

Angles

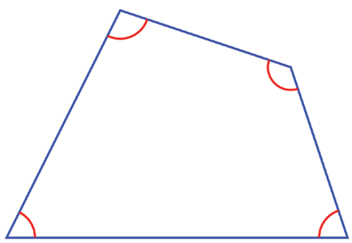
We have seen in class 7 that the sum of the three angles of any triangle is 180° (the section **Triangle sum** of the lesson **Parallel Lines**).

What about the sum of all four angles of a quadrilateral ?

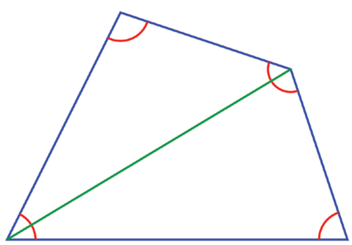
If it is a rectangle, each angle is 90° and so the sum of all four is $4 \times 90° = 360°$

Is this true for all quadrilaterals?

See this quadrilateral:



If we draw a diagonal, then two of its angles are split into two:



Strange polygons

Look at these figures:

These are also drawn using only straight lines. So sometimes, such figures are also called polygons. But in our lesson, we don't consider as polygons figures like these with corners turned inwards or sides crossing each other. It is because many of the general results we want to state do not hold for such figures.

The quadrilateral also is now split into two triangles. The top parts of the split angles are angles of the top triangle; the bottom parts of the angles are angles of the bottom triangle.

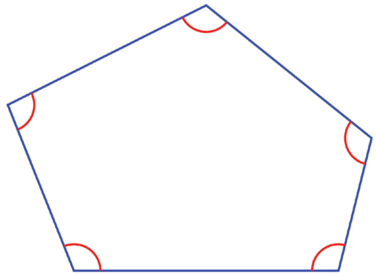
So if we add up all the angles of both the triangles, it amounts to adding all the angles of the quadrilateral.

The sum of the angles of each triangle is 180°; two triangles means twice this which is $2 \times 180° = 360°$.

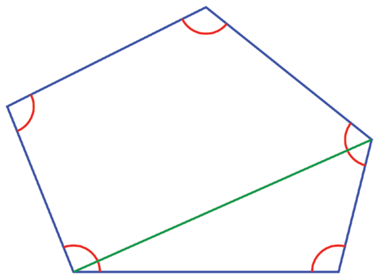
So what do we see here?

The sum of all the angles of a quadrilateral is 360°.

Next let's look at a pentagon:



Again, let's draw a diagonal like this:



(In any polygon, a line joining two non-adjacent vertices is called a diagonal).

Two of the angles of the pentagon are split into two by the diagonal; and the pentagon itself is split into a quadrilateral and a triangle.

The bottom parts of the split angles are angles of the triangle; and the top parts are angles of the quadrilateral.

So adding up the angles of the quadrilateral and the triangle, we get the sum of all five angles of the pentagon.

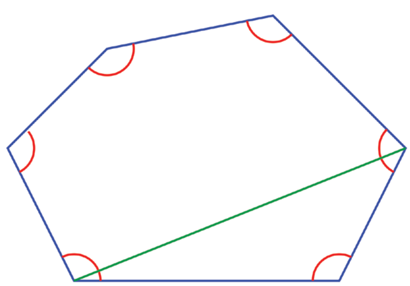
The sum of the angles of the quadrilateral is 360°

The sum of the angles of the triangle is 180°

The sum of the angles of the pentagon is 360° + 180° = 540°

Now can you guess the sum of the angles of a hexagon?

May be a picture will help:



We can think of a hexagon as the figure formed by joining a pentagon and a triangle.

The sum of the angles is 540° + 180° = 720°

So what can we say in general ?

- Taking the polygons in order as triangle, quadrilateral, pentagon and so on, when we move from one polygon to the next, the number of vertices and the number of sides increase by 1
- This change can be seen as adjoining a triangle
- The sum of the angles increase by 180°

So, we can write the sum of the angles in order as shown below:

	Polygon	**Sides**	**Sum of angles**
1	Triangle	3	180°
2	Quadrilateral	4	180° + 180° = 2 × 180°
3	Pentagon	5	(2 × 180°) + 180° = 3 × 180°
4	Hexagon	6	(3 × 180°) + 180° = 4 × 180°
5	Heptagon	....	....
6	Octagon	....	....

Looking at this pattern, can you say the sum of the angles of a polygon with 10 sides ?

We think like this:

(i) Taking triangle as the first polygon, quadrilateral as the second, pentagon as the third and so on, what is the position of the ten-sided polygon (decagon)?

(ii) So, continuing the table above, how many times 180° is the sum of the angles of a decagon?

Thus we can compute the sum of the angles of a 10-sided polygon as 8 × 180° = 1440°.

We can compute the sum of the angles of any polygon like this, can't we?

The sum of the angles of a polygon is 180° multiplied by two less than the number of sides

We can state this using a bit of algebra:

The sum of the angles of a polygon of n sides is $(n-2) \times 180°$

For example, how do we find the sum of the angles of a polygon of 20 sides ?

$$(20 - 2) \times 180° = 18 \times 180°$$
$$= (18^2 \times 10)°$$
$$= 3240°$$

A question in the other direction: is the sum of the angles of any polygon equal to 900° ?

In any polygon, the sum of the angles is a multiple of 180°, right?

Is 900 a multiple of 180?

To check this, we will have to divide 900 by 180.

First, we can remove the common factors (the section **Common factors** of the lesson **Within Numbers** in the class 5 textbook).

We can split 900 and 180 like this:

$$900 = 9 \times 10 \times 10$$
$$180 = 9 \times 10 \times 2$$

So that

$$900 \div 180 = 10 \div 2 = 5$$

So,

$$900° = 5 \times 180°$$

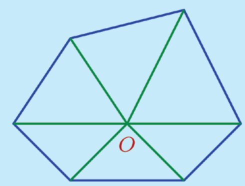
So there is a polygon with sum of the angles 900°.

And how many sides does it have?

5 + 2 = 7 sides, right?

Another division

We can divide a polygon into triangles in another fashion, by drawing lines from a point inside the polygon to the vertices.



An n-sided polygon gives n triangles like this, right? The sum of the angles of these triangles is

$$= n \times 180°$$

Among all these angles, those other than the angles at O add up to the sum of the angles of the polygon; and the angles at O add up to 360°, as we have seen earlier. Thus the sum of the angles of the polygon is

$$(n \times 180°) - (2 \times 180°)$$
$$= (n - 2) \times 180°$$

(1) The sum of the angles of a polygon is 1980°. What is the sum of the angles of a polygon with one side more ? And for a polygon with one side less?

(2) What is the sum of the angles of a 27-sided polygon?

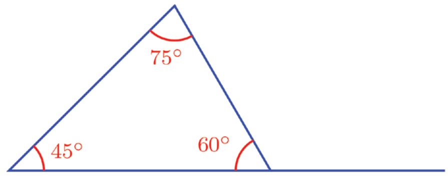
(3) The sum of the angles of a polygon is 8100°. How many sides does it have?

(4) Is the sum of the angles of any polygon equal to 1000°? Explain.

(5) A 20-sided polygon has equal angles. How much is each angle?

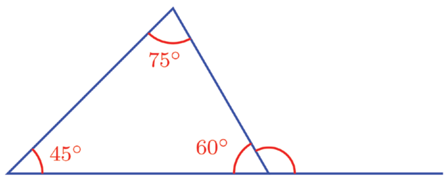
Outer angles

See this picture:



The bottom side of a triangle is extended a bit to the right.

Now there is an angle outside the triangle, at the right vertex:

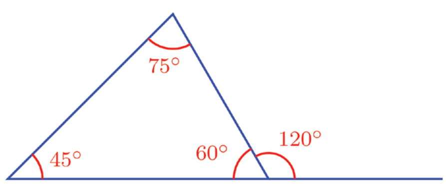


An angle formed outside the triangle at a vertex by extending a side is called an **outer angle** of the triangle. Then the angle inside the triangle at this vertex is called an **inner angle.**

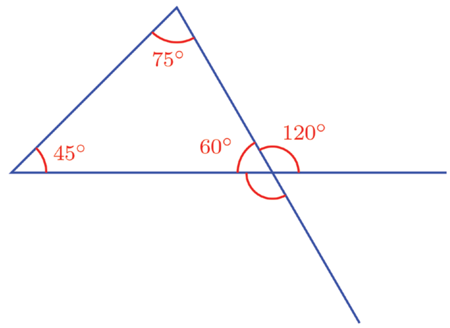
What is the measure of the outer angle in the picture?

The inner and outer angles are the angles formed on either side of the right side of the triangle meeting the extended bottom side. So, their sum is 180° (the section **When lines meet** of the lesson **Lines and Angles** in the class 6 textbook).

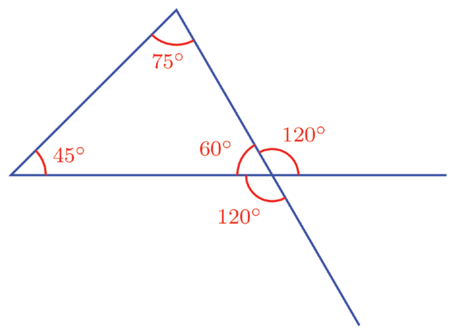
This means the outer angle is $180° - 60° = 120°$.



Now if the right side of the triangle is extended down, there would be another outer angle at the right vertex:

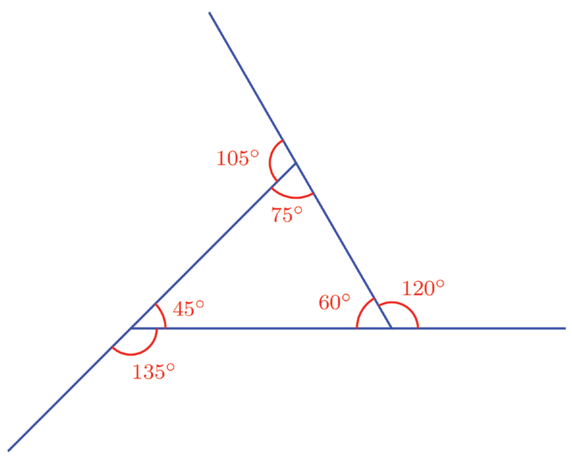


It also is 120°, isn't it? (Why?)



Thus we get the outer angle by subtracting the inner angle from 180°.

We can draw outer angles at the other two vertices also:



In any triangle, the sum of the inner angle and an outer angle at each vertex is 180°

We can see another thing here.

What is the sum of the inner angles at any two vertices?

For example, 45° + 60° = 105°

Isn't this equal to the outer angle at the third vertex?

Add other pairs of inner angles and check.

Isn't the sum of two inner angles at any two vertices equal to the outer angle at the third vertex? Why is this so?

Let's think:

(i) Since the sum of all three inner angles is 180°, the sum of two inner angles is equal to the third inner angle subtracted from 180°.

(ii) And this is equal to the outer angle at the third vertex.

The same reasoning holds, even if change the inner angles. That is, the sum of inner angles at any two vertices is equal to the outer angle at the third vertex.

We can also state it like this:

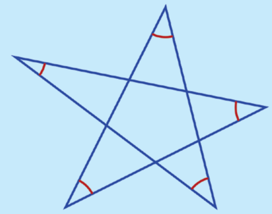
In any triangle, the outer angle at any vertex is equal to the sum of the inner angles at the other two vertices

One more thing. Add all six angles, three inner and three outer. We get 540°, right ? Draw triangles with other angles and check. Why is the sum 540° in all triangles ?

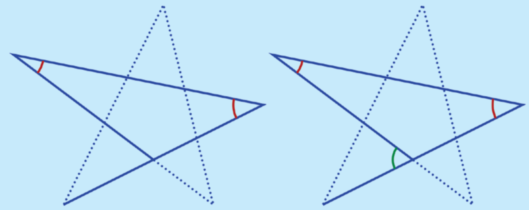
Split these six angles into three pairs of inner and outer angles at each vertex and add. What do you see?

Star math

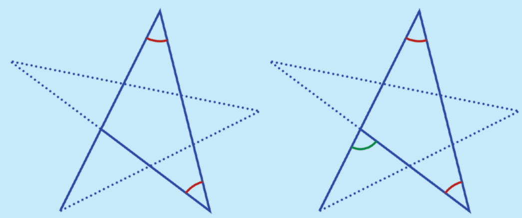
We want to find the sum of the angles at the five corners of this star:



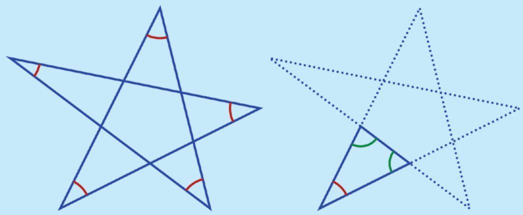
If we look only at the triangle shown in the first picture below, we can see that the sum of the angles marked in it is equal to the angle marked in green in the second picture:



Similarly, we can see the sum of the angles at two other corners of the star also as a single angle:



Thus the sum of the angles at the five corners of the star can be seen as the sum of three angles of a triangle:



Thus the sum of the angles at the corners of the star is 180°.

(i) The sum of each pair is 180°

(ii) The sum of all three pairs is 3 × 180° = 540°

(iii) The sum of all six angles is 540°

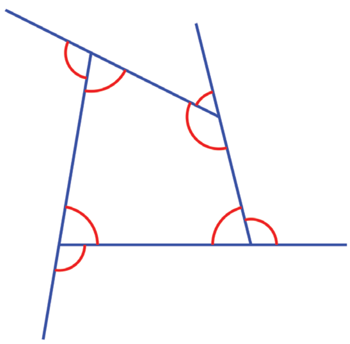
(iv) In this, the sum of the inner angles is 180°

(v) The sum of the outer angles is 540° – 180° = 360°

Whatever be the angles of the triangle, get the sum of the outer angles as 360°, by this reasoning.

In any triangle, if we take one outer angle at each vertex, then their sum is 360°

As in the case of triangle, we can extend the sides of a quadrilateral also to form outer angles:



Here also, the sum of an inner and an outer angle at each vertex is 180°, isn't it?

What is the sum of all four inner angles and all four outer angles?

If we pair the inner and outer angles at each vertex, as in the case of a triangle, we get four pairs here. And the sum of each pair is 180°; so the total sum is 4 × 180° = 720°

In this, the sum of the inner angles is 360°

The sum of outer angles is 720° – 360° = 360°

Thus in a quadrilateral also, the sum of outer angles is 360°.

What about a pentagon?

Let's think about without drawing a picture (or may be just visualizing it in mind). The sum of the inner angle and an outer angle at each vertex is 180°, and there are five such pairs; the sum of all these is 5 × 180°.

In this, the sum of the inner angles is (5 – 2) × 180° = 3 × 180°

So, the sum of outer angles is

$$(5 \times 180^\circ) - (3 \times 180^\circ) = (5-3) \times 180^\circ = 2 \times 180^\circ = 360^\circ$$

We can do such computations for any polygon.

In general, for a polygon of n vertices, if we pair the inner angle and an outer angle at each vertex, we can compute like this:

Sum of all angles is $n \times 180^\circ$

Sum of inner angles is $(n-2) \times 180^\circ$

The sum of outer angles is

$$(n \times 180^\circ) - ((n-2) \times 180^\circ) = (n-(n-2)) \times 180^\circ = 2 \times 180^\circ = 360^\circ$$

(Recall the section **Addition and Subtraction** of the lesson, **Shorthand Math** in the Class 7 textbook).

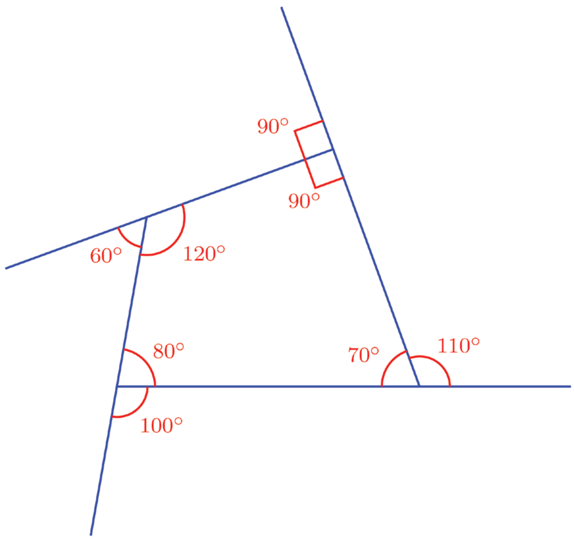
Thus,

If we take one outer angle at each vertex of any polygon, then the sum of these angles is 360°.

We can shorten this a bit:

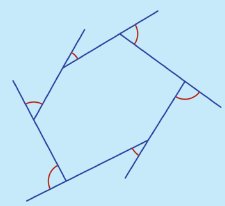
The sum of the outer angles of any polygon is 360°

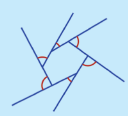
Let's do another computation using this. See this picture:

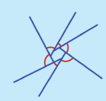


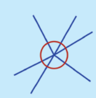
Shrinking, shrinking, ...

We can gradually shorten the sides of a polygon without changing any of its angles:









Finally, what if the polygon itself vanishes leaving a single point?

What about the sum of the external angles (outer angles) of the polygon?

Find the sum of outer angles at any two vertices and the sum of inner angles at the other two vertices.

For example, the sum of outer angles at the bottom is

$$100° + 110° = 210°$$

and the sum of the inner angles at the top is

$$120° + 90° = 210°$$

Check other pairs like these. Are the sums equal?

Is it true for other quadrilaterals also?

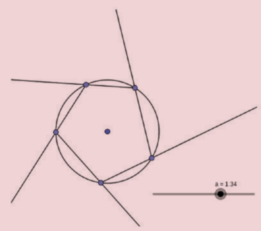
Let's take the inner angles of the quadrilateral as $a°$, $b°$, $c°$, $d°$ so that the outer angles are $180° - a°$, $180° - b°$, $180° - c°$, $180° - d°$.

Now consider the outer angles $180° - a°$ and $180° - d°$. Their sum is

$$(180° - a°) + (180° - d°) = 360° - (a° + d°)$$

The sum of all inner angles of the quadrilateral is 360°. That is

In GeoGebra, make a slider **a** with Min:0.01, Max:2 and Increment: 0.01. Draw a circle of radius **a** and mark five or six points on it. Join the points as shown below, using the Ray tool:



Now hide the circle and mark the outer angles. Change the value of **a** and see what happens.

$$a° + b° + c° + d° = 360°$$

So,

$$360° - (a° + d°) = b° + c°$$

Thus the sum of the two outer angles is equal to the sum of the inner angles at the other two vertices. We get the same result, whatever pairs of outer angles we choose.

The sum of the outer angles at any two vertices of a quadrilateral is equal to the sum of the inner angles at the other two vertices.

We have seen that in a triangle, the outer angle at any vertex is equal to the sum of the inner angles at the other two vertices; and in a quadrilateral, the sum of the outer angles at any two vertices is equal to the sum of the inner angles at the other two vertices.

So, here are some possible investigations:

(i) Is there any such relation between inner and outer angles of a pentagon?

(ii) And in a hexagon?

(iii) Is there a general relation which is true for all polygons?

(1) All inner angles of an 18-sided polygon are equal
 (i) How much is each outer angle?
 (ii) How much is each inner angle?

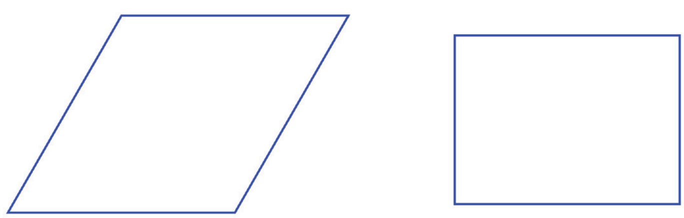
(2) (i) In which polygon is the sum of outer angles of a polygon, one from each vertex, equal to the sum of its inner angles.
 (ii) In which polygon is the sum of the outer angles twice the sum of the inner angles?
 (iii) In which polygon is the sum of the outer angles half the sum of the inner angles?
 (iv) In which polygon is the sum of the outer angles one-third the sum of the inner angles?

Regular polygons

We have seen that if the sides of a triangle are of the same length, then the size of its angles also are the same; and on the other hand, if the angles of a triangle are of the same size, then the sides are of the same length (the section **Isosceles triangles** of the lesson **Equal Triangles**).

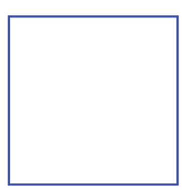
But in a general polygon, even if the sides are of the same length, the angles may not be of the same size; and even if the angles are of the same size, the sides may not be of the same length.

For example, any rhombus has sides of the same length, but the angles may be different; and any rectangle has all angles are right; but the lengths of the sides may be different:



And in a square?

Sides are equal and so are the angles.



As mentioned at the beginning of the lesson, polygons with the lengths of sides equal and sizes of angles equal are called **regular polygons**. We have seen that the sum of the angles of an n-sided polygon is $(n-2)\times 180°$ and the sum of the outer angles is 360°. In a regular polygon angles are all equal; and so the outer angles are also equal (why?). So, we can compute each inner and outer angle of a regular polygon:

In a regular polygon of n sides

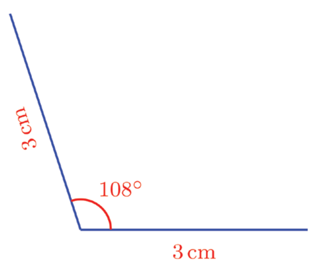
- **each inner angle is** $\frac{n-2}{n} \times 180^\circ$
- **each outer angle is** $\frac{1}{n} \times 360^\circ$

For example, in a regular pentagon, each inner angle is

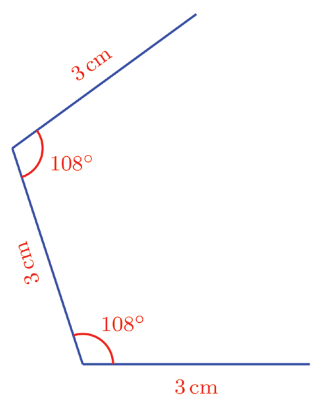
$$\frac{3}{5} \times 180^\circ = 108^\circ$$

So, how do we draw a regular pentagon with each side 3 centimetres?

First draw a line 3 centimetres long and another line of 3 centimetres at one of its ends, making an angle of 108°.

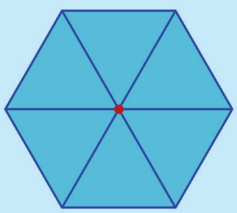


Next draw a 3 centimetre long line at the other end of the second line, again making an angle of 108°:



Joining together

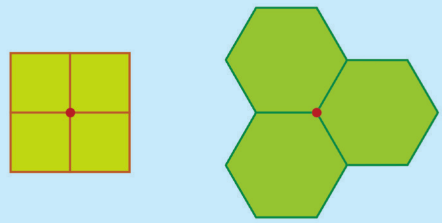
See how six equal equilateral triangles join at a point:



What other equal regular polygons can be joined around a point like this?

The sum of angles around a point should be 360°, isn't it? So if equal, regular polygons are to be joined around a point, the size of an angle of the polygon should be a factor of 360

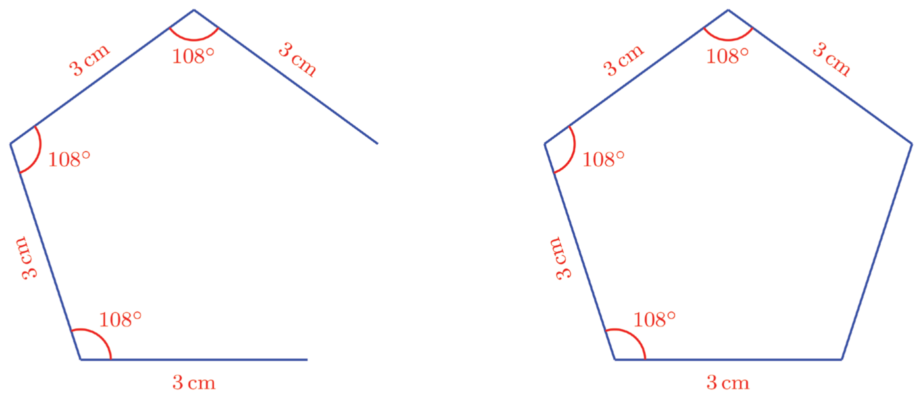
See these pictures:



Can we do this with any other regular polygons?

What if they are not regular polygons?

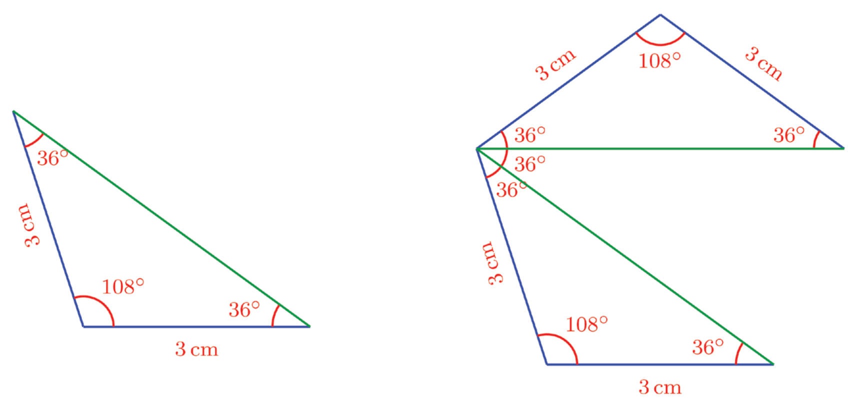
Draw another line of 3 centimetres, making an angle of 108° with this line and join its other end to the end of the first line:



We have a pentagon now; is it regular?

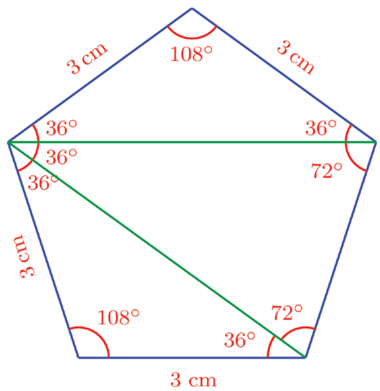
How can we be sure that the length of the last line is also 3 centimetres long and that the last two angles formed are also 108°?

Let's look at the angles first. We can draw an isosceles triangle with the first two lines as sides and another isosceles triangle with the third and the fourth lines as sides:



In these triangle the lengths (3 centimetres) of two (blue) sides and the angle between them (108°) are equal. So, the lengths of their third (green) sides are also equal (the section **Two sides** of the lesson **Equal Triangles**).

Thus the last line drawn is a side of an isosceles triangle and we can compute its angles:



Thus we see that the last two angles formed are also 36° + 72° = 108°

Compass

We can draw some regular polygons using compass instead of using a set square or protractor to draw angles. We have seen in earlier classes how equilateral triangles, squares and hexagons can be drawn like this.

There are several ways to draw a regular pentagon using compass. A simple method can be found in the web page

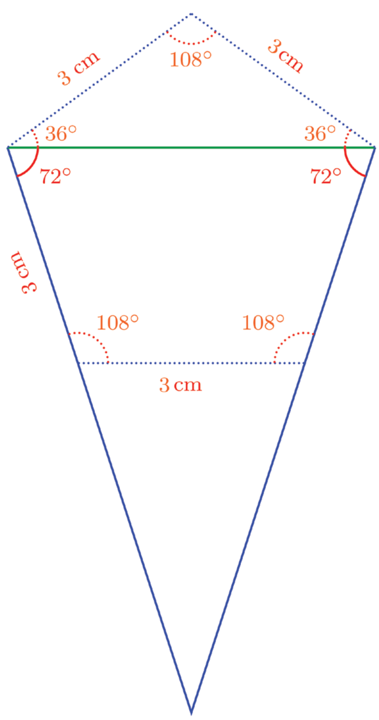
www.cut-the-knot.org/pythagoras/PentagonConstruction.shtml

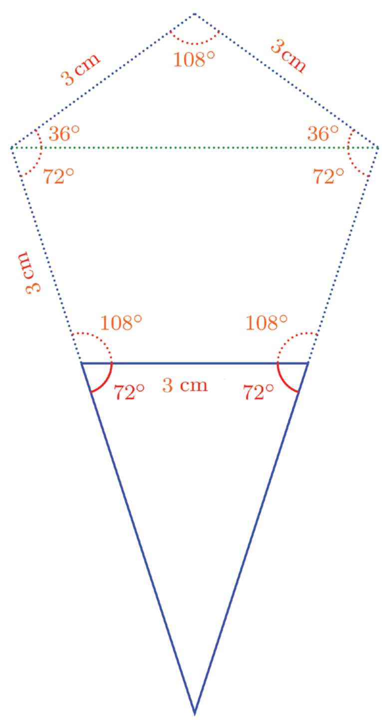
The famous mathematician Gauss proved (when he was just 19) that a regular pentagon of 17 sides can be drawn using ruler and compass.

Details of this can be found in

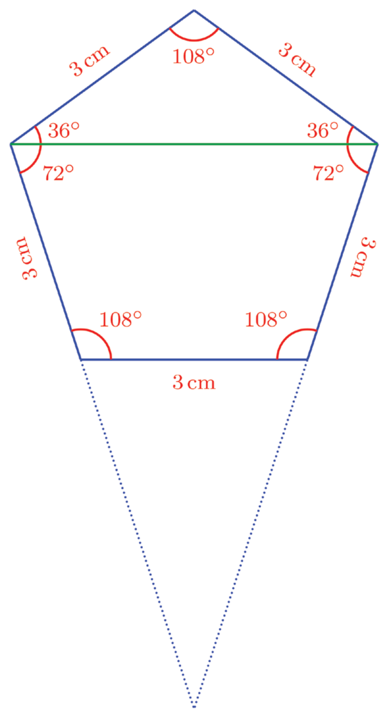
en.wikipedia.org/wiki/Heptadecagon

Now to see that the length of the last line is also 3 centimetres, extend the left and right sides of the pentagon to meet at a point. Then we get two isosceles triangles (how?).

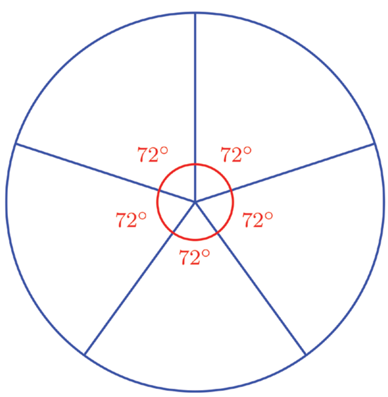


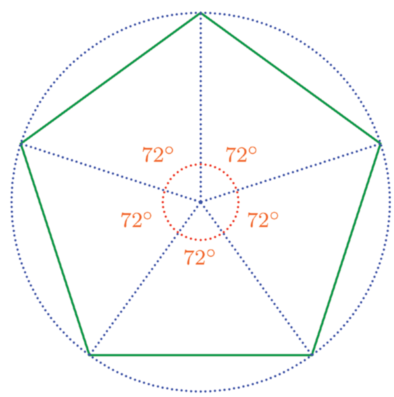


The numbers got by subtracting the length of equal sides of the small triangle from those of the large triangle must also be equal. The difference in lengths of the left sides is 3 centimetres. So, the difference in lengths of the right sides must also be 3 centimetres:



There is another way to draw a regular pentagon. Draw a circle and draw five angles at the centre, each being $\frac{1}{5} \times 360° = 72°$:





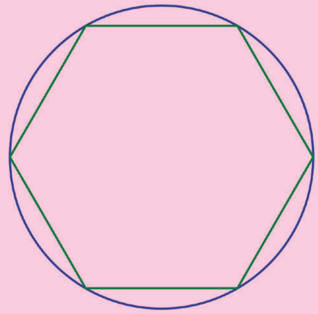
Can you explain why in this pentagon the sides are of the same length and angles of the same size?

One problem with this method of drawing regular pentagons is that it is not easy to calculate what the radius of the circle should be, to get a regular pentagon of sides 3 centimetres.

(1) (i) Draw a hexagon of equal sides with angles different.

(ii) Draw a hexagon of equal angles with different sides.

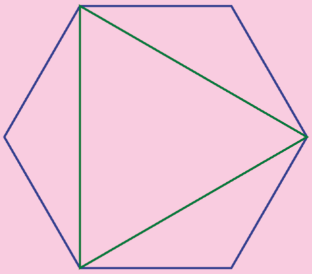
(2) The picture shows a regular hexagon with vertices on a circle. Prove that the length of its sides is equal to the radius of the circle.



(3) Draw a regular octagon (eight sides) of sides 3 centimetres.

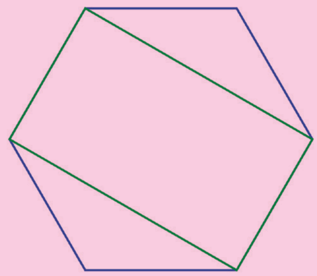
(4) Draw a circle of radius 3 centimetres and draw a regular octagon with all vertices on this circle.

(5) The picture below shows a regular hexagon and a triangle joining its alternate vertices:



Is this an equilateral triangle? Why?

(6) The picture below shows a regular hexagon and a quadrilateral joining four of its vertices:



Is this quadrilateral a rectangle ? Why ?

(7) Calculate an inner angle and an outer angle of a regular polygon of 15 sides.

(8) An outer angle of a regular polygon is 20°. How many sides does it have?

(9) An inner angle of a regular polygon is 168°. How many sides does it have?

5

Solutions of Equations

Adding and subtracting

Zuhra opened her money box and started counting her savings. "How much is there?", asked her mother. "If you give me seven rupees more, I would have round fifty", Zuhra said hopefully.

How much does she have now?

7 more rupees would make it 50; which means what she has is 7 less than 50

$$50 - 7 = 43$$

(1) "Six more marks and I would have got hundred out of hundred in my math exam", Rajan thought sadly. How much did he score?

(2) "5 more years, and I would be 18. I can vote", Lissy calculated. How old is she now?

(3) What should be added to 123 to make it 321?

(4) What should be subtracted from 432 to get 234?

Another problem:

Unni spent 8 rupees from what he got for vishu to buy a pen. Now he has 42 rupees left. How much did he get for vishu?

When it decreased by 8 rupees, it became 42 rupees. So what he got is 8 more than 42.

$$42 + 8 = 50$$

(1) Gopalan bought a bunch of bananas. 7 of the bananas had begun to rot and he removed them. Now there are 46. How many bananas were there in the bunch?

(2) Vimala bought some things for 163 rupees and now she has 337 rupees left. How much did she have at first?

(3) What number becomes 321 on subtracting 123?

Multiplication and division

In an investment scheme, the deposited amount would be doubled after 6 years. To get ten thousand rupees finally, how much should be deposited now?

10000 is double the deposit; so the deposit is half of 10000, which is 5000.

Four persons divided equally the profits they got from their business and Jose got 1500 rupees. What is the total profit?

1500 rupees is $\frac{1}{4}$ of the profit.

So, the total profit is 4 times 1500, which is $4 \times 1500 = 6000$ rupees.

(1) The salary of the manager of an office is five times the salary of a peon. The manager gets 75000 rupees a month. How much does the peon get?

(2) Some friends went for a trip and decided to divide equally among them the 12500 rupees they had spent. Each had to pay 2500 rupees. How many were there in the group?

(3) A number multiplied by 12 gives 756. What is the number?

(4) A number divided by 21 gives 756. What is the number?

Different kinds of change

Look at this problem:

46 rupees is spent in buying two notebooks and a pen. The price of the pen is six rupees. What is the price of a notebook?

We think like this:

The total cost of 46 rupees includes the price of a pen which is 6 rupees. What if the pen was not bought?

The cost would have been only 40 rupees.

This 40 rupees is the price of two notebooks. So, the price of a notebook is 20 rupees.

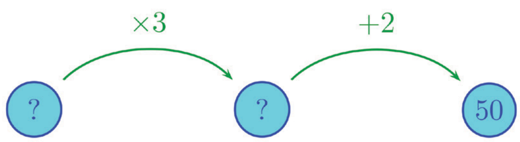
Now let's do this computation in reverse:

Two note books of 20 rupees makes 40 rupees; the price of the pen is 6 rupees. 46 rupees altogether.

Now look at this problem:

> Two added to three times a number makes 50. What is the number?

An unknown number multiplied by three and two added to the product makes 50.



Ancient Math

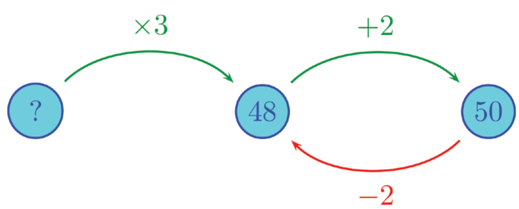
Ancient Egyptians used to keep written records, as early as 3000 BCE. In those times, the material used for writing was the flattened stem of a plant names Papyrus. Archaeologists have unearthed a large number of such manuscripts. Such manuscripts are also called papyri (plural of papyrus).

Such a papyrus from Egypt discusses some mathematical problems and their solutions. It is estimated to be written about 1650 BCE. It says at the beginning that it is copied from a two hundred year old work by the scribe named Ahmos. This is now in the British Museum, and is referred to as the Ahmos Papyrus. (Since it was discovered by Alexander Rhind, it is also called the Rhind Papyrus).

The problems discussed are about numbers and geometric figures

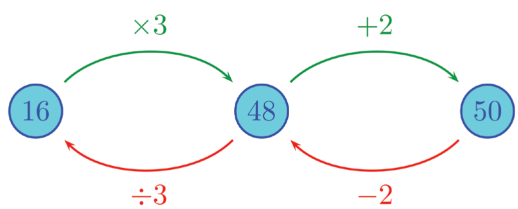
To get back the original number, what all things should we do?

The number became 50 on the final addition of 2; so it was $50 - 2 = 48$ before doing this:



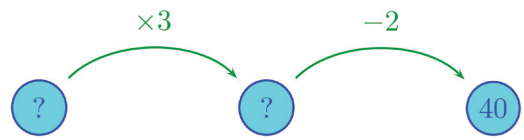
Now how do we get back to the original number from 48?

The number became 48 on multiplication by 3; so before that it was $48 \div 3 = 16$

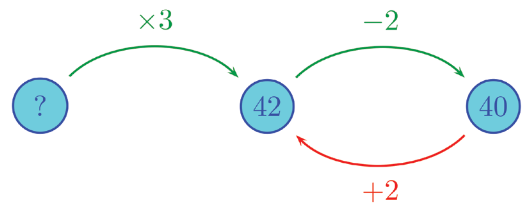


Let's change this problem like this:

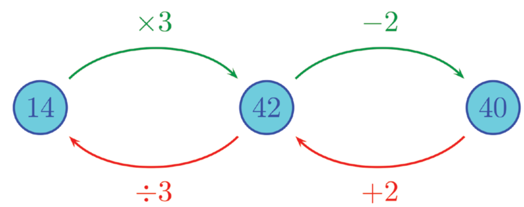
Two subtracted from three times a number gives 40. What is the number?



Here the number before the subtraction of 2 was $40 + 2 = 42$;



This was got on multiplication by3; so before that it was $42 \div 3 = 14$



Now such problems can be done in head, right?

Look at this problem:

2 added to a number and the sum multiplied by 3 gives 60. What is the number?

We think like this:

(i) It became 60 on the final multiplication by 3; so it was $60 \div 3 = 20$ before this

(ii) It became 20 on adding 2; so, it was $20 - 2 = 18$ before that.

(1) The perimeter of a rectangle is 50 metres and the length of one side is 5 metres. What is the length of the other side?

(2) When Anita and her friend bought 5 pens together, they got a reduction of 3 rupees in the price. The total cost was 32 rupees. Had they bought the pens individually, how much would it have cost each?

(3) In each of the problems below, the result of doing some operations on a number is given. Find each number:

(i) Three added to two times the number gives 101

(ii) Two added to three times the number gives 101

(iii) Three subtracted from two times the number gives 101

(iv) Two subtracted from three times the number gives 101

(4) The sum of 6 times a number and 4 is 100. What is the number?

(5) 4 times the sum of a number and 6 gives 100. What is the number?

Multiple and part

A small problem:

3 notebooks for Gopu and 2 for his sister Meenu together cost 100 rupees. What is the price of a notebook?

Altogether 5 notebooks are bought and the total price is 100 rupees.

So, the price of one notebook is 100 ÷ 5 = 20 rupees.

Let's state this problem using only the number indicating the price of a notebook:

3 times a number and 2 times the same number added together makes 100. What is the number?

3 times any number and 2 times the same number makes 5 times the number, doesn't it?

Thus 5 times the number is 100 and the number is $\frac{1}{5}$ of 100, which is 20

Let's look at another problem with just numbers:

10 added to 3 times a number makes it 5 times the number. What is the number?

Let's think:

(i) 5 times the number is 10 more than 3 times the number

(ii) The difference of 3 times the number from 5 times the number is 10

(iii) 2 times the number is 10

(iv) The number is half of 10, which is 5

Inverse operations

If we know the result of *2 added to a number*, then to find the number, we must *subtract* 2. What if we know the result of subtracting 2 from a number, then to get the number back, we must add 2.

Similarly, to recover a number from its product with 2, we must take the *quotient* of the result by 2, and to recover a number from its quotient by 2, we must find the product of the result with 2.

The mathematician Bhaskara of India, discusses these in his work, **Lilavati**. He calls it the "Inverse Operation Method" and describes it thus:

If we know the result, then to get the number, change division to multiplication, multiplication to division, square to square root square root to square.

(1) The sum of 2 times a number and 7 times the same number is 27. What is the number ?

(2) 99 added to 12 times a number gives 21 times the number. What is the number?

Let's look at another problem:

A full bottle of water and a quarter of it makes one litre. How much does the bottle hold?

A full bottle and a quarter of it makes $1\frac{1}{4} = \frac{5}{4}$ of the bottle. This is said to be 1 litre.

That means $\frac{5}{4}$ times the water in the bottle is 1 litre.

So, the amount of water that the bottle can hold is $\frac{4}{5}$ of 1 litre (the section **Topsy-turvy** of the lesson **Reciprocals** in the class 7 textbook).

That is, the bottle can hold $\frac{4}{5}$ litre, which is $\frac{4}{5} \times 1000 = 800$ millilitre.

This problem can also be stated simply as a problem of numbers:

If $\frac{5}{4}$ of a number is 1, then what is the number?

and we can compute the number as $\frac{4}{5} \times 1 = \frac{4}{5}$

Another problem:

A bucket is one-third filled with water. One more litre of water poured into it makes it half full. How much water can the bucket hold?

Using only the number indicating the amount of water that the bucket can hold, the question can be written like this:

1 added to $\frac{1}{3}$ of a number makes it $\frac{1}{2}$. What is the number?

Here 1 is the difference of $\frac{1}{3}$ of the number from $\frac{1}{2}$ of the number.

What fraction of the number is this difference?

$$\frac{1}{2} - \frac{1}{3} = \frac{3}{6} - \frac{2}{6} = \frac{1}{6}$$

Thus $\frac{1}{6}$ of the number is 1 and so the number is 6

The bucket can hold 6 litres of water.

Here's an old riddle that is a part of the local folklore:

A child asked a flock of birds, "how many are you?" and a bird replied,

"We and us again,

With half of us

And half of that

With one more

Would make a hundred"

How many birds in the flock?

Let's do the operations the bird said on the number of birds:

We and us again	:	2 times the number
With half of us	:	$2\frac{1}{2}$ of the number
And half of that	:	$2\frac{3}{4}$ of the number
With 1 more	:	1 added to $2\frac{3}{4}$ of the number

So, the problem becomes this

1 added to $2\frac{3}{4}$ of the number is 100. What is the number?

First we compute $2\frac{3}{4}$ of the number as $100 - 1 = 99$. Then we write $2\frac{3}{4}$:

$$2\frac{3}{4} = 2 + \frac{3}{4} = \frac{8}{4} + \frac{3}{4} = \frac{11}{4}$$

This means $\frac{11}{4}$ of the number is 99

So, the number is

$$\frac{4}{11} \times 99 = \frac{9 \times 11 \times 4}{11} = 9 \times 4 = 36$$

Thus there are 36 birds in the flock.

(1) A third of a piece of rope cut away leaves 10 metres. What was the original length?

(2) Half the milk in a can and a third of the remaining was used up and now there is 5 litres remaining. How many litres did it originally contain?

Algebraic methods

What is the general nature of the problems we have done so far?

The result of doing some operations on an unknown number is given, and the problem is to find the original number we started with.

How did we find the number?

Do the inverse of the operation done on the unknown number in the reverse direction (that is, the operation done last inverted first) on the given number.

For example, look at this problem:

> Rashida spent 200 rupees to buy 4 kilograms of okra and coriander and curry leaves for 20 rupees. What is the price of a kilogram of okra?

First we write it in mathematical language:

> 20 added to 4 times a number makes 200. What is the number?

How do we find the number?

First subtract the 20 added last and then divide by the 4, by which it was multiplied first. That is,

$$(200 - 20) \div 4 = 180 \div 4 = 45$$

Thus we find that the price of one kilogram of okra is 45 rupees.

Now look at this problem:

> A 10 metre long wire is to be bent into a rectangle, with the length 1 metre longer than the width. What should be the lengths of the sides?

The perimeter of the rectangle is 10 metres and it is twice the sum of the length and width. Here length is 1 metre longer than the width, so that the sum of the length and width is the sum of the width and 1 more than the width.

So, how do we write the problem using only numbers?

> The sum of a number and 1 more than the number, when doubled gives 10. What is the number?

If we eliminate the final doubling, the problem can be put like this:

The sum of a number and 1 more than the number is 5. What is the number?

Recall seeing in class 7 that the sum of a number and one more than the number is equal to one added to twice the number (the section **Numbers and algebra** of the lesson **Algebra**).

And we noted that it is more convenient to write it using algebra,

$$x + (x + 1) = 2x + 1 \text{ for all numbers } x$$

We can use this in our current problem. If we take the number we are seeking in this problem as x, the problem becomes this:

If $2x + 1 = 5$ then what is x ?

What does this mean?

1 added to 2 times a number is 5. What is the number?

We can find the number through inverse operations:

$$(5 - 1) \div 2 = 2$$

Thus we get the width and length of the rectangle as 2 metres and 3 metres.

Sometimes it is more convenient to do such problems using algebra from the very beginning.

See this problem:

85 rupees was spent in buying pens for the 25 children in a class. There were 4 rupee pens and 3 rupee pens. How many of each was bought ?

The total number of pens bought is 25. So, if we can find the number of one kind of pens, then the number of the other kind can be found by subtracting it from 25.

First let's try to find the number of 4 rupee pens. How can we rewrite the problem using only this number?

What's in a name?

Algebra came to Europe during the Renaissance, through the translation of mathematical texts in Arab.

Many such were the works of the Arab mathematician Muhammad al Khwarizmi.

He lived during the eighth century CE.

Given that 2 subtracted from a number gives 5, we find the number by adding 2 to 5. Al-Khwarizmi calls such operations, al-jabr, meaning "to join" or "to restore". The word "algebra" in English is derived from this Arab word.

A step-by-step procedure to solve a problem (especially in computer programming) is called an "algorithm". This word is derived from the name "Al-Khwarizmi”

The sum of 4 times a number, and 3 times the number subtracted from 25 is 85. What is the number?

This is difficult to untangle using inverse operations, isn't it?

Let's convert the problem into algebra.

We take the number of 4 rupee pens as x and write the various numbers in the problem in terms of x:

The number of 4 rupee pens	:	x
Their price	:	$4x$
The number of 3 rupee pens	:	$25 - x$
Their price	:	$3(25 - x)$
Total price	:	$4x + 3(25 - x)$

So, what is the question ?

If $4x + 3(25 - x) = 85$ then what is x ?

The expression $3(25 - x)$ in this can be written as

$$3(25 - x) = (3 \times 25) - (3 \times x)$$
$$= 75 - 3x$$

Using this, we can change $4x + 3(25 - x)$ also:

$$4x + 3(25 - x) = 4x + 75 - 3x$$
$$= 4x - 3x + 75$$
$$= x + 75$$

So how can we write our problem now?

If $x + 75 = 85$, then what is x ?

If 75 added to a number makes it 85, then the number is 10, isn't it ?

So in this problem, $x = 10$

That is, the number of 4 rupee pens is 10 and the number of 3 rupee pens is $25 - 10 = 15$

(1) The perimeter of a rectangle is 80 metres and its length is one metre more than twice the width. What are its length and width?

(2) When a hundred rupee note was changed to twenty-rupee and ten-rupee notes, seven notes were got. How many of each?

(3) The price of a book is 4 rupees more than the price of a pen. The price of a pencil is 2 rupees less than the price of this pen. A person bought 5 books, 2 pens and 3 pencils and paid 54 rupees for them. What is the price of each?

(4) A square containing four numbers in a calendar is marked and the sum of these numbers is 80. What are the numbers?

Let's look at another problem:

Ajayan is ten years older than Vijayan. Next year, Ajayan's age will be twice that of Vijayan, What are their ages now?

Let's start with algebra. Taking Vijayan's age now as x, we can write the numbers in the problem like this:

Vijayan's age	:	x
Ajayan's age	:	$x + 10$
Vijayan's age next year	:	$x + 1$
Ajayan's age next year	:	$x + 11$

It is said that Ajyan's age next year will be twice Vijayan's age next year.

So, how do we rewrite the problem?

If $2(x + 1) = x + 11$ what is x ?

We can write the expression $2(x + 1)$ as

$$2(x + 1) = 2x + 2$$

So, how do we write the problem?

If $2x + 2 = x + 11$ then what is, x ?

Equations

1 added to 2 times a number gives 9. How do we write this using algebra ?

$$2x + 1 = 9$$

We can easily see that the number in question is 4. In other words, the equation above is true only if we take x as 4. We say that

$x = 4$ is the solution of the equation $2x + 1 = 9$

We have seen that some algebraic equations are true whatever be the numbers we take as the letters in it. For example,

$$x + (x + 1) = 2x + 1$$

$$(x + y)^2 = x^2 + y^2 + 2xy$$

$$x - (y - z) = (x - y) + z$$

Such equations are called **identities.**

What next ?

There are different ways to proceed

We can think like this:

(i) 2 added to the number $2x$ gives the number $x + 11$

(ii) $2x$ is got by subtracting 2 from $x + 11$

(iii) $2x = (x + 11) - 2$

(iv) $2x = x + 9$

What does the last equation mean?

9 added to the number x gives $2x$; which means x is doubled.

A number is doubled when it is added to itself, right?

So, x is the number 9

Thus Vijayan's age is 9 and Ajayan's age is $9 + 10 = 19$

Speciality of 9

Take any two-digit number ending in 9 and add together the sum and product of the digits.

For example, if we take 29, then sum of digits is 11 and the product of digits is 18.

And the sum of these is $11 + 18 = 29$

Is this true for all two-digit numbers ending in 9 ?

Take the number as $10x + 9$ and see.

Does two-digit numbers ending in any number other than 9 has this speciality?

From the equation,

$$x + y + xy = 10x + y$$

what do we get as y?

There is another way to do this. The algebraic form of the problem is this:

If $2x + 2 = x + 11$ then what is x ?

The meaning of this equation is that for some number x, the operations $2x + 2$ and the operation $x + 11$ give the same result.

So, we must get the same result, if the same number is subtracted after these operations, right ?

What if we subtract x?

$$(2x + 2) - x = x + 2$$
$$(x + 11) - x = 11$$

And we have seen that these must be equal; that is,

$$x + 2 = 11$$

From this, can't we see that $x = 9$?

See this problem:

A solution of acid in water contains 60% acid. When 6 litres of acid was added to it, there was 75% acid. How many litres of acid did the original solution contain ?

Here the total quantity of the solution at the beginning is not mentioned. Let's take it as x litres. It is said that 60% of it is acid and 40% water. That is,

Solution	x litres
Acid	$\frac{60}{100}x = \frac{3}{5}x$ litres
Water	$\frac{40}{100}x = \frac{2}{5}x$ litres

How do these change when 6 more litres of acid is added ?

Solution	$x + 6$ litres
Acid	$\frac{3}{5}x + 6$ litres
Water	$\frac{2}{5}x$ litres

It is said the solution now is 75% acid; that is, the amount of acid now is

$$(x + 6) \times \frac{75}{100} = \frac{3}{4}(x + 6) \text{ litres}$$

The quantity of acid now was first calculated as $\frac{3}{5}x + 6$ litres.

$$\text{So, } \frac{3}{4}(x+6) = \frac{3}{5}x + 6$$

This we can write as

$$\frac{3}{4}x + 4\frac{1}{2} = \frac{3}{5}x + 6$$

This means $4\frac{1}{2}$ added to the number $\frac{3}{4}x$ makes it the number $\frac{3}{5}x + 6$. So, the first number is $4\frac{1}{2}$ subtracted from the second number, right?

$$\frac{3}{4}x = \left(\frac{3}{5}x + 6\right) - 4\frac{1}{2}$$

$$= \frac{3}{5}x + 1\frac{1}{2}$$

What does this mean?

$\frac{3}{4}x$ is $1\frac{1}{2}$ more than $\frac{3}{5}x$. In other words, the difference of $\frac{3}{4}x$ and $\frac{3}{5}x$ is $1\frac{1}{2}$. That is

$$\frac{3}{4}x - \frac{3}{5}x = 1\frac{1}{2}$$

We can calculate $\frac{3}{4}x - \frac{3}{5}x$ like this:

$$\frac{3}{4}x - \frac{3}{5}x = \frac{3}{20}x$$

So, the equation of our problem becomes

$$\frac{3}{20}x = 1\frac{1}{2}$$

This means $\frac{3}{20}$ of the number x is $1\frac{1}{2}$. So, the number x is $\frac{20}{3}$ times $1\frac{1}{2}$, isn't it? (The section **Times and parts** of the lesson **Reciprocals** in the class 7 textbook).

$$\begin{aligned} x &= \frac{20}{3} \times 1\frac{1}{2} \\ &= \frac{20}{3} \times \frac{3}{2} \\ &= \frac{20 \times 3}{3 \times 2} \\ &= 10 \end{aligned}$$

So, originally there was 10 litres of solution.

(1) The age of Appu's mother is now 9 times that of Appu. After nine years, it will be three times Appu's age. What are their ages now?

(2) A class has the same number of girls and boys. On a day when eight boys were absent, the number of girls was twice the number of boys. What are the number of girls and the number of boys?

(3) In a class of girls and boys, 50% of the children are girls. When 10 more girls were admitted to this class, this became 60%. How many children were in the class at first?

(4) Another problem from folk-math. Some lotus flowers have bloomed in a pond. A flock birds sat on the flowers to rest. First, one bird sat on each flower, but one bird didn't have a flower to sit. Then two birds sat on each flower and there was one flower extra. How many lotuses and how many birds?

6

New Numbers

Coloured numbers

Neetu, Hari and Anvar are playing a card game: 50 cards with numbers 1 to 5 on them, 10 of each number; half the numbers in black and the other half in red.

Each player starts with a black 5. The remaining cards are shuffled and kept stacked in the middle, face downwards. Now each draws a card in turn from the stack. If the number is black it is added; if it is red, then the number is subtracted. The game continues. The first to cross 10 wins.

The first draw was this:

Neetu 2 Anvar 1 Hari 3

And so the scoreboard was this:

Neetu	5	7
Anvar	5	6
Hari	5	2

The second draw was this:

Neetu 1 Anvar 3 Hari 3

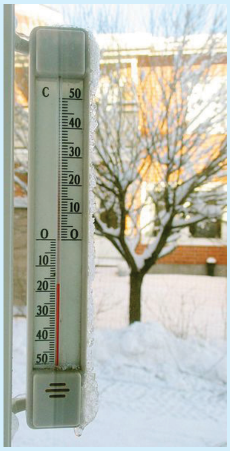
Negative temperature

You might have seen daily weather bulletins in the newspapers and TV. During winter, the temperatures at many places in the northern parts of India are given as –1°C or –2° C. What does this mean?

The temperature at which water begins to freeze is taken as zero degree Celsius (0° C). Temperatures below this are indicated with a minus sign.

A column of mercury enclosed with a narrow glass tube expands and rises when the temperature rises; and contracts and falls when the temperature is lowered. Temperature is usually measured using such a device, called a thermometer.

Thermometers used in countries experiencing extremely cold climates have numbers marked below zero. The thermometer in the picture shows temperature between –20°C and –15°C.



How to write the points?

Neetu	5	7	8
Anvar	5	6	3
Hari	5	2	

What to do with Hari's score, they wondered. Since 3 cannot be subtracted from 2, Hari said, his score should be written as 0.

No, said Anvar; Hari has lost and only he and Neetu should continue the game.

Let Hari continue and 1 point subtracted from what he gets next, said Neetu.

The others agreed and they decided to write "subtract 1" in Hari's cell.

Why not just –1, asked Anvar and the others agreed to this:

Neetu	5	7	8
Anvar	5	6	3
Hari	5	2	–1

Hari was lucky in the next draw:

Neetu 4 Anvar 5 Hari 3

How to write the scores now?

Neetu	5	7	8	4
Anvar	5	6	3	
Hari	5	2	–1	

Colder and colder

The coldest place in India is Drass, a town in Ladakh. Temperature as low as –60° C was recorded here.

The coldest place in the world is Antarctica, where the lowest temperature recorded is –89° C

Hari now got 3 points

If we subtract the penalty of 1 from the last round, it becomes 2.

What about Anvar?

5 cannot be subtracted from 3. As in the case of Hari earlier, they decided to subtract from his next score.

Subtract how many points?

Instead of "subtract 2", they decided to write –2

Neetu	5	7	8	4
Anvar	5	6	3	–2
Hari	5	2	–1	2

This was the draw in the fourth round:

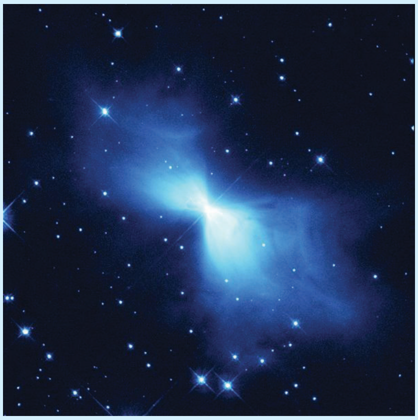
Neetu 3 Anvar 2 Hari 5

Can you fill up the scoreboard?

Neetu	5	7	8	4	
Anvar	5	6	3	–2	
Hari	5	2	–1	2	

Limit of coldness

In the whole of the universe that we know, the coldest temperature is found in the Boomerang Nebula, about 50000 trillion (5×10^{16}) kilometres away from the earth. It is –272.15°C



Even though this is the coldest temperature in nature, colder temperatures can be artificially created in labs.

However, temperature at or below –273.15° C cannot occur naturally nor created artificially, according to the laws of temperature in physics.

Below zero

In the card game, when 3 had to be subtracted from 2, we wrote it as –1. This can be written

$$2 - 3 = -1$$

What does this mean?

2 subtracted from 2 gives 0. What is to be subtracted is 3; so we subtract 1 more and write it as –1

$$0 - 1 = -1$$

And how did we subtract 5 from 3?

3 subtracted from 3 gives 0; how much more should be subtracted?

$$0 - 2 = -2$$

Such numbers written with a minus sign are called **negative numbers.**

Let's look at another problem:

There are 25 questions in a test. Each correct answer gets 2 marks. For each wrong answer, 1 mark is subtracted.

For example, if 19 answers are correct and 6 wrong, the marks would be

$$(19 \times 2) - 6 = 32$$

What if it's the other way round?

Negative amounts

From the seventh century CE onwards, we see negative numbers being used in India, to denote debts. This practice is still continued in some transactions.

The 6 correct answers would get $(6 \times 2) = 12$ marks; and for the 19 wrong answers, 19 marks have to be subtracted.

Thus the marks is $12 - 19$

How do we compute this ?

12 subtracted from 12 gives 0; how much more should be subtracted ?

$$19 - 12 = 7$$

So,

$$12 - 19 = 0 - 7 = -7$$

Sometimes we need negative fractions also.

Look at this problem:

> There are 10 questions in a test. Each correct answer gets 1 mark; for each wrong answer, $\frac{1}{2}$ is subtracted. A person answers all questions, but only 3 of them correct. How much marks does he get ?

For the 3 correct answers, they get 3 marks; for the 7 wrong answers, half of 7, that is $3\frac{1}{2}$ has to be subtracted.

3 subtracted from 3 gives 0. $\frac{1}{2}$ has to be subtracted. So, total marks is

$$3 - 3\frac{1}{2} = 0 - \frac{1}{2} = -\frac{1}{2}$$

In this test, how much would a person with only one correct answer get?

$$1 - 4\frac{1}{2}$$

How do we compute this?

$$1 - 1 = 0$$

How much more to subtract?

$$4\frac{1}{2} - 1 = 3\frac{1}{2}$$

So,

$$1 - 4\frac{1}{2} = 0 - 3\frac{1}{2} = -3\frac{1}{2}$$

Once we start using negative numbers, other numbers such as 1, 2, $1\frac{1}{2}$ (which are not negative) are called **positive numbers**.

What about the number 0? It is considered neither positive nor negative.

To compute the difference of a positive number from a smaller one, we first reduced it to zero and then subtracted from zero. Can't we do it directly?

Look at the various computations done earlier:

$2 - 3 = -1$	$3 - 2 = 1$
$3 - 5 = -2$	$5 - 3 = 2$
$12 - 19 = -7$	$19 - 12 = 7$
$3 - 3\frac{1}{2} = -\frac{1}{2}$	$3\frac{1}{2} - 3 = \frac{1}{2}$
$1 - 4\frac{1}{2} = -3\frac{1}{2}$	$4\frac{1}{2} - 1 = 3\frac{1}{2}$

What do you see here?

Subtracting a positive number from a smaller positive number gives the negative of the number got by subtracting the smaller from the larger

Negative floors

In tall buildings, lifts are used to move from one floor to another. It has buttons with the numbers of the floors, which are pressed to go to a specific floor. The picture shows such panel of buttons:

Why are there buttons with −1 and −2 in this?

This building has some floors below the ground level. The first of these is denoted −1, the second −2 and so on.

We can write this using algebra also:

$$x - y = -(y - x) \text{ for all } 0 < x < y$$

(1) Calculate the following

(i) $4 - 9$ (ii) $14 - 29$ (iii) $5 - 10$

(iv) $25 - 65$ (v) $\frac{1}{2} - \frac{3}{4}$ (vi) $\frac{1}{3} - \frac{1}{2}$

Addition and subtraction

In the game with number cards, one's points is –2 means, 2 points has to be subtracted from what he gets later. If he gets a black 2 from the stack in the next round, his points become

$$2 - 2 = 0$$

Getting 2 points when one has –2 points can also be written as

$$-2 + 2$$

Thus

$$-2 + 2 = 2 - 2 = 0$$

In a test of 10 questions, 1 mark is given for each correct answer and 1 mark subtracted for each wrong one.

If one gets the first 5 wrong, but the next 5 right, what would be his score?

If from the 5 marks for the correct answers, 5 marks are subtracted because of the wrong ones, total marks is 0

If valued in the order of the questions, we can write the marks as –5 + 5. Thus

$$-5 + 5 = 5 - 5 = 0$$

What if the first 4 answers are wrong, but the next 6 correct?

We can write it as –4 + 6. If from the 6 marks for the correct answers, 4 marks are subtracted because of the wrong answers, then it is 6 – 4 = 2. Thus

$$-4 + 6 = 6 - 4 = 2$$

Direction change

In describing motion along a straight line, the distances from a fixed point on the line are denoted by positive numbers in one direction and negative numbers in the opposite direction:

In the picture above distance to the right of the red dot are taken as positive and distances to the left of the dot are taken negative.

If an object travels 3 metres to the right from the dot and then 5 metres to the left, would it be finally on the left or right of the dot? At what distance from it?

We can write this as

$$3 - 5 = -2$$

If the travel is 5 metres to the left first and then 3 metres to the right?

$$-5 + 3 = -2$$

What if it is 5 metres to the left first and then 3 metres to the left again?

What if the first 6 are wrong and the next four right? We can write the total marks as $-6 + 4$

If from the 4 marks for the correct answers, 6 marks are subtracted due to the wrong answers, we get $4 - 6 = -2$. So

$$-6 + 4 = 4 - 6 = -2$$

Suppose that in a test of 10 questions, 1 mark is given for each correct answer and $\frac{1}{2}$ mark is subtracted for each wrong answer. If one gets only the last 3 answers correct how much marks would they get?

We can compute the total marks as $3 - 3\frac{1}{2} = -\frac{1}{2}$ as seen before. If marks are calculated in the order of the question numbers, we can say that the total marks is $-3\frac{1}{2} + 3$. Thus

$$-3\frac{1}{2} + 3 = 3 - 3\frac{1}{2} = -\frac{1}{2}$$

Let's look at all these computations together:

$$-2 + 2 = 2 - 2 = 0$$

$$-5 + 5 = 5 - 5 = 0$$

$$-4 + 6 = 6 - 4 = 2$$

$$-6 + 4 = 4 - 6 = -2$$

$$-3\frac{1}{2} + 3 = 3 - 3\frac{1}{2} = -\frac{1}{2}$$

What do we see from this?

Adding the negative of a positive number to a positive number means, subtracting the first number from the second

Using algebra, we can state it like this:

$-x + y = y - x$ for all positive numbers x and y

Calculate the following:

(i) $-4 + 9$ (ii) $-9 + 4$ (iii) $-15 + 8$

(iv) $-8 + 15$ (v) $-\frac{1}{2} + \frac{3}{4}$ (vi) $-\frac{3}{4} + \frac{1}{2}$

Speed math

As an object thrown upwards from the earth goes upwards, its speed decreases every instant. When the speed decreases to zero, it begins to fall down with increasing speed till it hits the ground.

If it is thrown straight up, speed decreases at the rate of 9.8 m/s every second. For example, if it is thrown up straight with a speed of 49 m/s, its speed would be $49 - 9.8 = 39.2$ m/s at the end of one second, $49 - (2 \times 9.8) = 29.4$ m/s at the end of two seconds and so on.

At the end of 5 seconds, speed would be

$$49 - (5 \times 9.8) = 0$$

Then it starts falling down with speed increasing at the same rate of 9.8 m/s. What would be the speed at end of 7 seconds from the start?

It is $7 - 5 = 2$ seconds since it has started falling. So, speed has increased 2×9.8 m/s from zero. That is 19.6 m/s.

Let's condense this using algebra: what is the speed at the end of t seconds from the start?

If $t < 5$, then

$$49 - 9.8\,t \text{ m/s}$$

What if $t > 5$? It is $t - 5$ after it has started the downward journey.

So speed is $(t - 5) \times 9.8$m/s,

which is $9.8t - 49$ m/s

Subtracting again

In a test where 1 mark is subtracted for every wrong answer, if the first 2 answers are wrong, what would be the marks then?

If the next answer is also wrong?

Since all 3 answers are wrong, the marks would be −3, right?

We can look at it another way: since the first 2 answers are wrong, marks at that stage is −2; and since the next answer is also wrong 1 more mark has to be subtracted, that is −2 − 1 Thus

$$-2 - 1 = -3$$

What if the next 2 answers are also wrong?

5 wrong answers means −5 marks; this can also be seen as subtracting 2 from −3; that is −3 − 2

So, we have

$$-3 - 2 = -5$$

> **Negative speed**
>
> We wrote the speed of an object thrown straight up with a speed of 49 m/s, using two algebraic equations:
>
> If $t < 5$, then $v = 49 - 9.8t$
>
> If $t > 5$, then $v = 9.8t - 49$
>
> If we denote the speeds upwards by positive numbers and the speeds downwards by negative numbers, then we need only one equation to compute the speed at any time t, which is
>
> $v = 49 - 9.8t$
>
> For example, the speed at the end of 8 seconds from start is
>
> $49 - (9.8 \times 8) = -29.4$ m/s

So, what is −5 − 3 ?

−5 is 5 less than 0; if it is decreased by 3 more? What is the total decrease? That is

$$-5 - 3 = -(5 + 3) = -8$$

Can't you calculate −5 − 7 like this?

$$-5 - 7 = -(5 + 7) = -12$$

In general

If from the negative of a positive number, we subtract a positive number, we get the negative of the sum of these positive numbers

This can be stated using algebra like this:

$-x - y = -(x + y)$ for any two positive numbers x and y

Do the computations below:

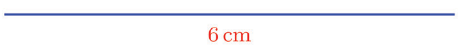
(i) −3 − 1	(iv) −7 − 8	(vii) 8 − 12	(x) $-\frac{1}{2} - \frac{1}{4}$
(ii) −9 + 4	(v) −1 − 1	(viii) $1\frac{1}{2} - 7\frac{1}{2}$	(xi) $-2\frac{1}{2} - 1\frac{1}{2}$
(iii) −10 − 4	(vi) −10 + 20	(ix) $-25 - 3\frac{1}{2}$	(xii) $-3\frac{1}{2} + 3\frac{1}{2}$

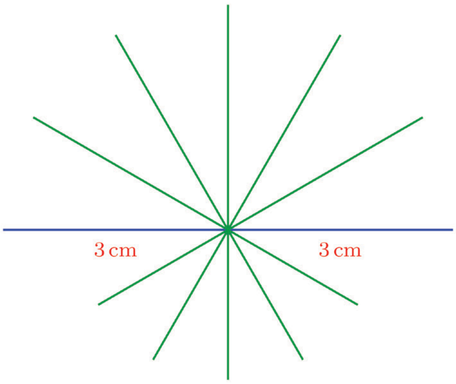
7

BISECTORS

Bisectors of lines

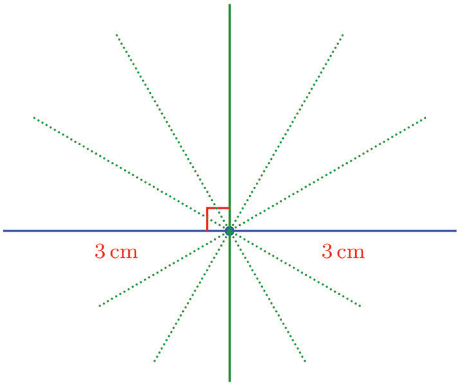
See these pictures:





All lines which divide a line into two equal parts are called **bisectors** of that line.

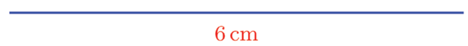
Only one of them is perpendicular to the line also:

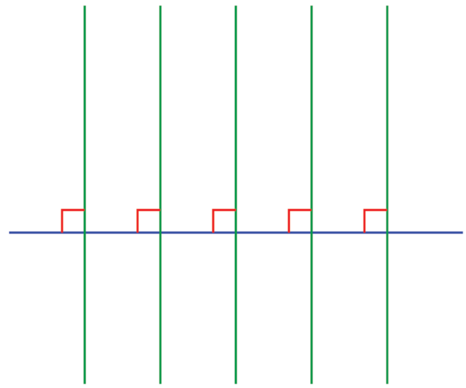


This bisector is called the **perpendicular bisector** of the line.

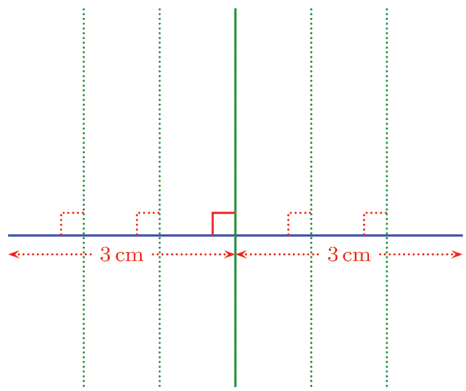
This can be put another way.

We can draw a perpendicular to a line through any point on it:

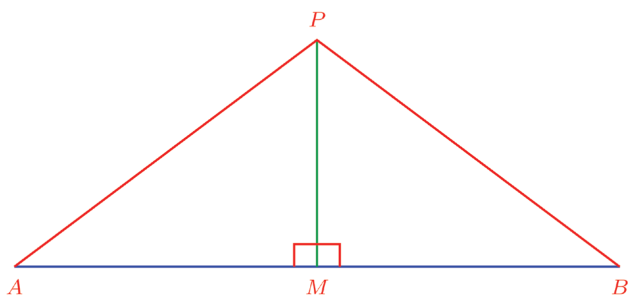




And only one of these lines is through the midpoint of the line; and it is the **perpendicular bisector** of the line:



Now look at this picture. The ends of the line *AB* are joined to a point *P* on the perpendicular bisector:



The lines *AP* and *BP* seem to be equal, don't they?

Let's check whether it is true.

AP and *BP* are the hypotenuses of the right triangles *AMP* and *BMP*.

So by Pythagoras' Theorem,

$$AP^2 = AM^2 + MP^2$$

$$BP^2 = BM^2 + MP^2$$

(The section, **Rectangle and square** of the lesson **Squares and right triangles** in the class 7 textbook).

What can we say about the lengths *AM* and *BM* in these equations?

Since *PM* bisects *AB*, we have $AM = BM$.

So, what about *AP* and *BP*?

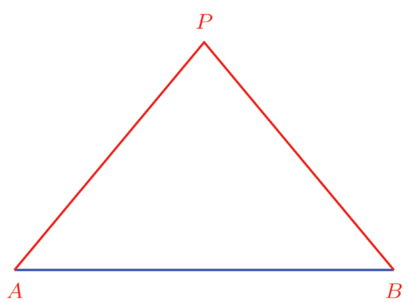
$$AP = BP$$

Let's write what we have seen as a general result.

Any point on the perpendicular bisector of a line is at the same distance from its endpoints

This raises a question: is this true the other way round?

That is, if a point is at the same distance from the endpoints of a line, does it lie on the perpendicular bisector of that line?



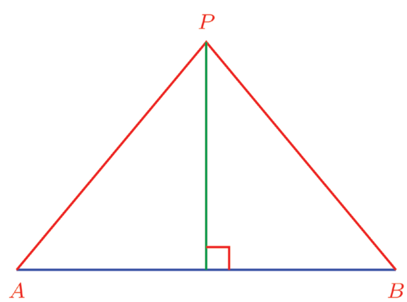
If we select the Perpendicular Bisector tool in GeoGebra and click on a line *AB* we get its perpendicular bisector. Mark a point *C* on this bisector, join *AC* and *BC*, and mark their lengths. Move *C* on the bisector and check these lengths.

In the picture, *P* is a point at the same distance from the ends *A* and *B* of the line *AB*.

We want to check whether *P* is on the perpendicular bisector of *AB*.

We can slightly rephrase our question: does the perpendicular from *P* to *AB* bisect *AB*?

Lets draw this perpendicular:



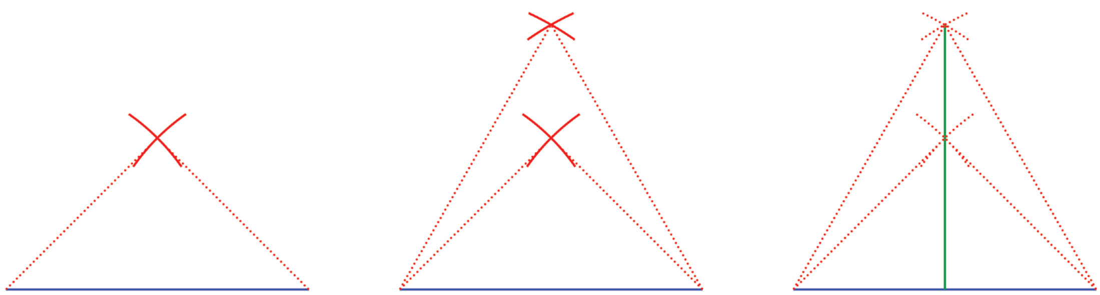
The point P is at the same distance from A and B; that is $AP = PB$. So, APB is an isosceles triangle. In this triangle, the perpendicular from the vertex P where the equal sides meet, to its opposite side AB, bisects this side (the section **Isosceles triangles** of the lesson **Equal Triangles**).

Thus the perpendicular from P is the perpendicular bisector of AB and so, the result stated above is true the other way round also.

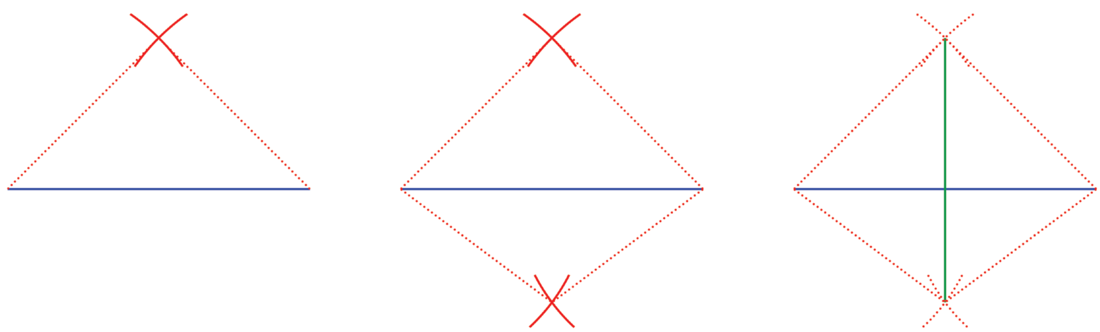
Any point at the same distance from the endpoints of a line lies on its perpendicular bisector

According to this result, if we choose two points which are at equal distances from the endpoints of a line, these two points would be on the perpendicular bisector of that line. So, the perpendicular bisector of the line is the line joining these two points (To draw a line, we need just two points on it, right?).

In GeoGebra, make a slider **a** with Min:0, Max:10, Increment:0.01. Draw a line AB and draw circles of radius **a** centered at A and B. Mark the points of intersection of these circles and select the option **Trace On** by right clicking them. Now right click on the slider and select Animation On. See what we get.



The points marked may be on either side of the line:



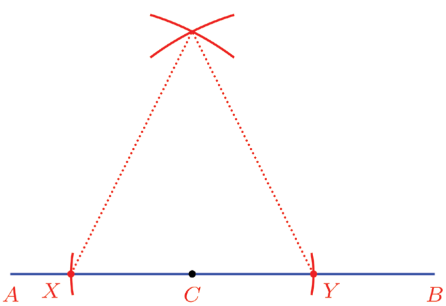
We can use this to draw the perpendicular to a line through a specified point also, without using a set square or protractor:

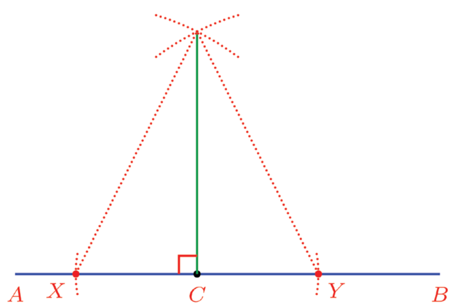
For example, suppose we want to draw the perpendicular to the line AB in the picture below, through the point C on it:

If we mark off a small part of *AB* with *C* as midpoint and draw the perpendicular bisector of this part, it would be perpendicular to *AB* also, isn't it?

For that first mark points *X* and *Y* on *AB* on either side of *C*, at the same distance from it:

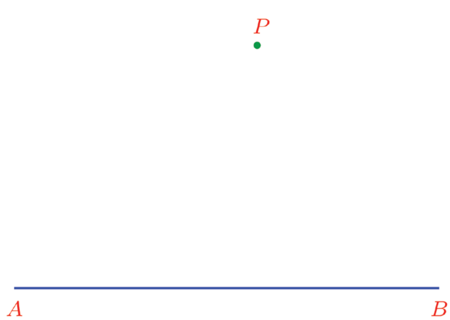
Now *C* is the midpoint of *XY*, so that the perpendicular bisector of *XY* passes through *C*. To get another point on this perpendicular bisector, we need only mark a point at the same distance from *X* and *Y*. Now we can draw a perpendicular line to AB through C; isn't it?

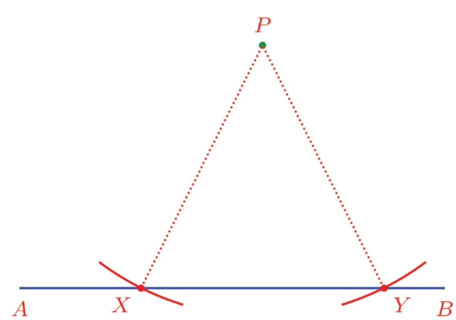




There is a similar method to draw the perpendicular to a line from a point not on it.

First mark two points on the line at the same distance from the point:

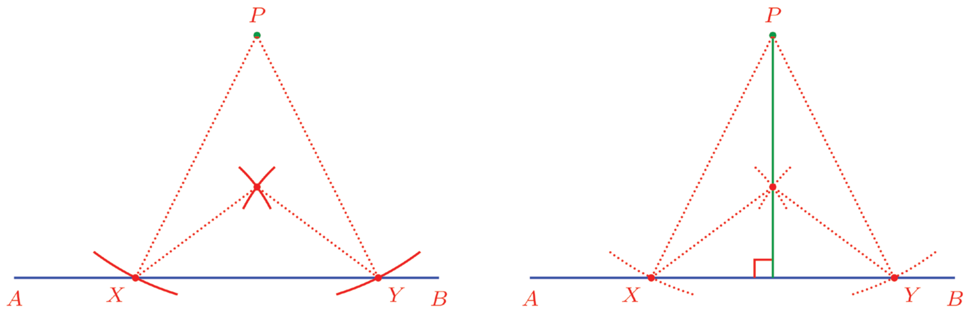




In the picture above, *X* and *Y* are points on *AB* at the same distance from *P*.

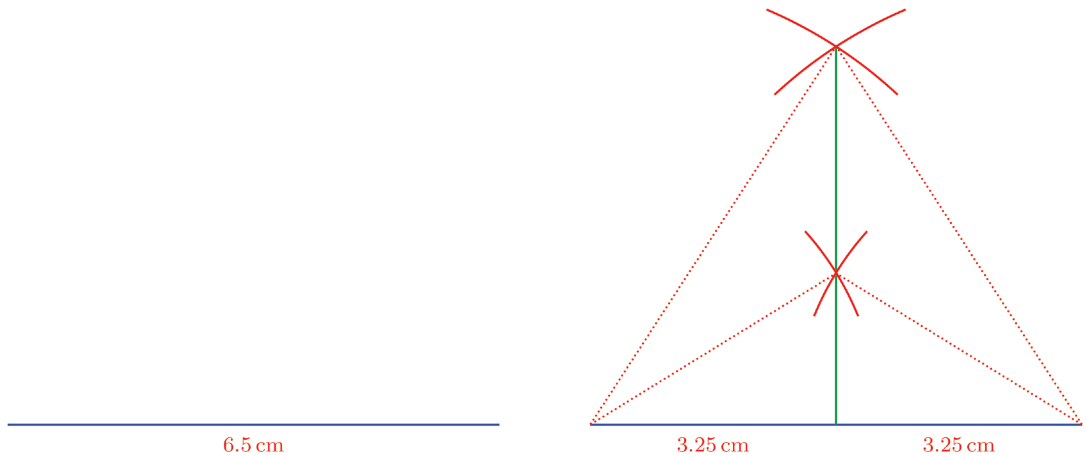
So, P is a point on the perpendicular bisector of *XY*.

If we mark one more point at equal distances from *X* and *Y*, we would get another point on the perpendicular bisector of *XY*. Draw a line passing through this point and *P*. And this bisector is perpendicular to *AB* also:

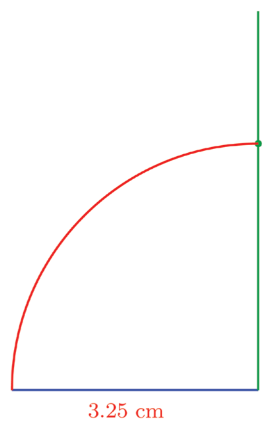


Now let's see how we can draw a square of side 3.25 centimetres, using these.

Drawing a line 6.5 centimetres long and its perpendicular bisector gives us a line 3.25 centimetres long:

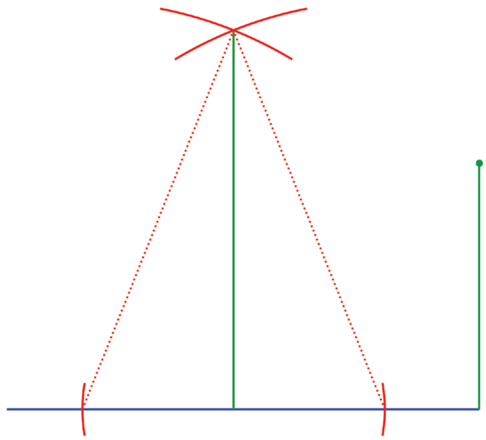


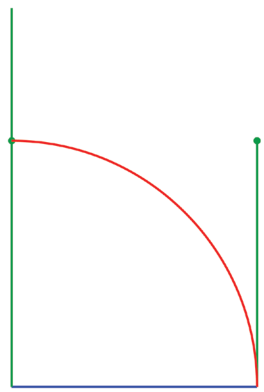
Marking half the length of the line on this bisector gives us two sides of the square:



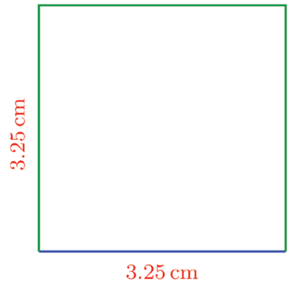
(At this point, we can use a protractor or set square to draw perpendiculars and to get the triangle, as done in earlier classes; here we draw without such instruments).

To get the third side, we draw the perpendicular from the other end of the bottom side and mark the length of the side on it also. (To draw this perpendicular, we will have to extend the bottom side a bit to the left):





Now joining the top ends of the perpendiculars gives us the square (why?).



Note that we used only ruler and compass to draw this square.

Draw the figures below using only ruler and compass

(1) Square of sides $4\frac{1}{4}$ centimetres

(2) Rectangle of sides $5\frac{1}{4}$ centimetres and $3\frac{1}{4}$ centimetres

(3) Equilateral triangle of side 2.75 centimetres

(4) Triangle of area 9 square centimetres and one side 4.5 centimetres

Rope math

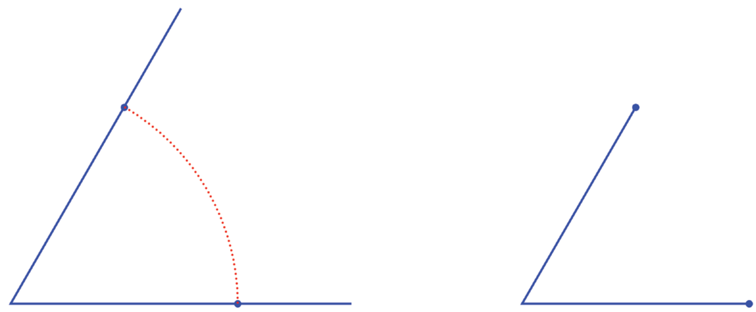
You might have heard about the book, Elements, the definitive book on ancient geometry. In it, Euclid considers only figures that can be drawn using lines and circles. In other words, only figures that can be drawn with an unmarked ruler (called a straight-edge) and a compass.

Why is this so? During the ancient ages, ropes or strings were used to measure and draw. Lines and circles are the only figures we can draw using rope. A piece of rope stretched between two pegs gave a line; if one is pulled up and rotated around the other, we get a circle.

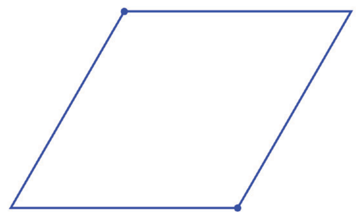
Now when instruments can be made to draw various figures, such constructions using only straight-edge and compass have only academic or historical value.

Rhombuses

Draw an angle and mark the same distance on its sides:



Next draw the line parallel to the horizontal line through the marked point on the top, and the line parallel to the slanted line through the marked point at the bottom to make a quadrilateral:

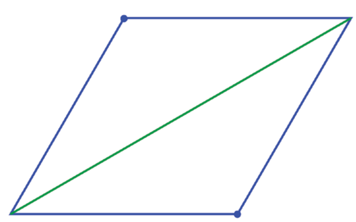


This is a parallelogram, isn't it? So, the opposite sides are of the same length (the section **Two angles** of the lesson **Equal Triangles**).

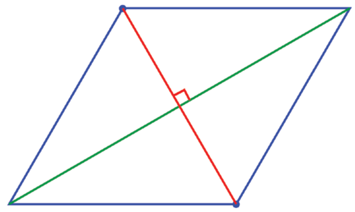
The first two lines drawn are of the same length. Thus this parallelogram has equal sides.

A parallelogram with all sides of the same length is called a **rhombus**.

Look at one of its diagonals. It joins two vertices and the distance from these two vertices to the other two vertices are equal:



So, the other two vertices are on the perpendicular bisector of the diagonal; in other words, the second diagonal, which joins the other two vertices, is the perpendicular bisector of the first diagonal.



In the same way if we start by drawing the shorter diagonal first, we can see that the longer diagonal is its perpendicular bisector.

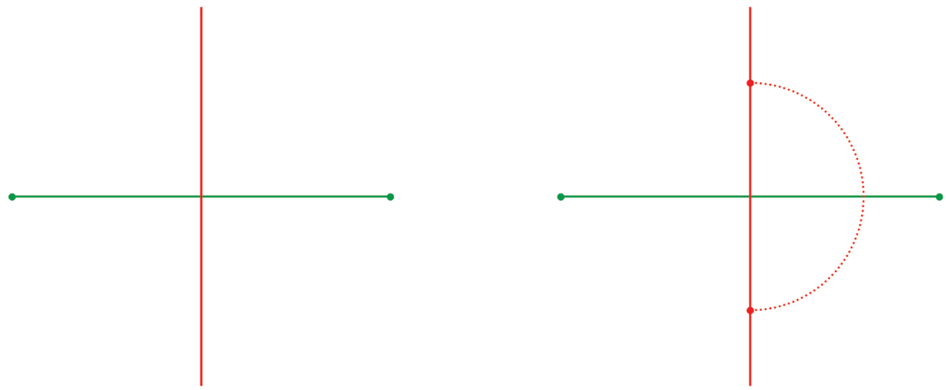
We state this as a general result:

In any rhombus, the diagonals are perpendicular bisectors of each other

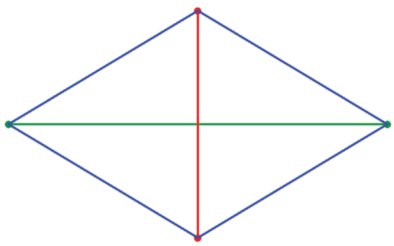
There is an immediate question: if we draw a quadrilateral with its diagonals perpendicular bisectors of each other, would it be rhombus ?

First let's draw such a quadrilateral

For that, draw a line and its perpendicular bisector. Mark equal distances from the midpoint on the bisector on both sides:



Joining these points to the endpoints of the first line gives us a quadrilateral in which diagonals are perpendicular bisectors of each other:



The question is whether it is a rhombus.

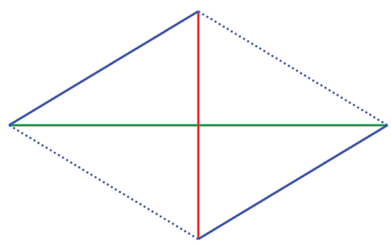
Since the diagonals are perpendicular to each other, they divide the quadrilateral into four right triangles.

Since the red line bisects the green line, the green sides of the triangles are of the same length; and since the green line bisects the red line, the red sides of the triangles are of the same length.

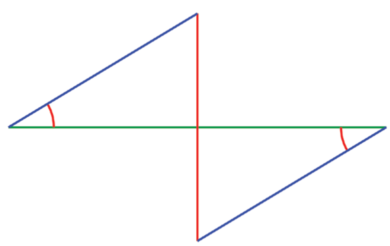
So, the blue lines, which are the hypotenuses of these four right triangles are also of the same length, by Pythagoras' Theorem.

In other words, all sides of this quadrilateral are of the same length.

Now we must check whether the pairs of opposite sides of this quadrilateral are parallel. First let's look at the top-left and bottom-right sides:



To see whether these are parallel, we need only check the angles marked in the picture below:



Since the lengths of the sides of the triangles in the picture are the same, these angles are equal; and so the blue lines are parallel.

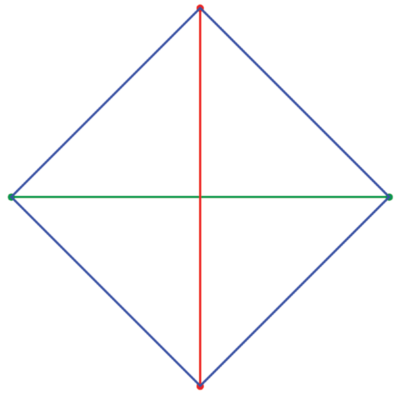
Thus in the quadrilateral one pair of sides are equal and parallel; and so, it is a parallelogram (the section **Two sides** of the lesson **Equal Triangles**).

What did we see here?

If the diagonals of a quadrilateral are perpendicular bisectors of each other, then the quadrilateral is a rhombus

What if the diagonals are also equal?

All four right triangle making up the rhombus become isosceles and so the angle at each vertex becomes 90°. This makes the rhombus a square:



So, here's a new way to draw a square.

Draw a square with diagonals 6 centimetres.

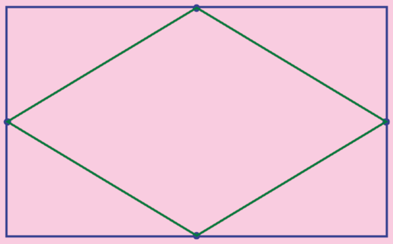
(1) Draw a rhombus with each pair of lengths given below for the diagonals:

(i) 6 centimetres, 4 centimetres

(ii) 6.5 centimetres, 4 centimetres

(iii) 6 centimetres, 4.5 centimetres

(2) The picture shows the quadrilateral formed by joining the midpoints of a rectangle:



(i) Are the diagonals of this quadrilateral parallel to the sides of the rectangle? Why?

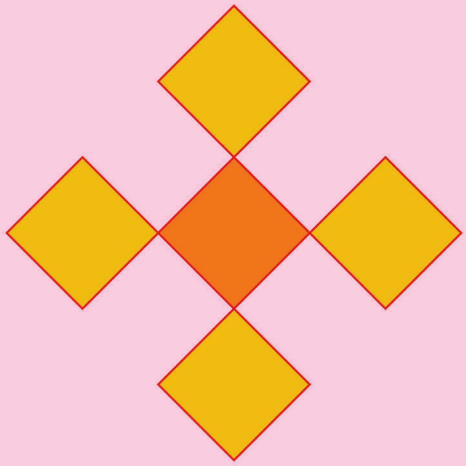
(ii) Is this quadrilateral a rhombus? Why?

(3) Draw a rhombus with diagonals 6.5 centimetres and 4.5 centimetres.

(4) Prove that each diagonal of a rhombus bisects the angles at the vertices it joins.

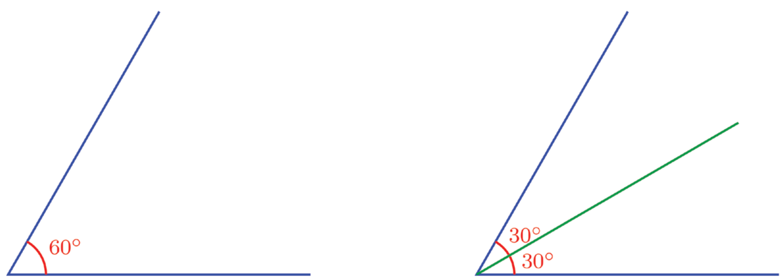
(5) Draw a square with diagonals 7 centimetres.

(6) Draw this picture



Bisector of an angle

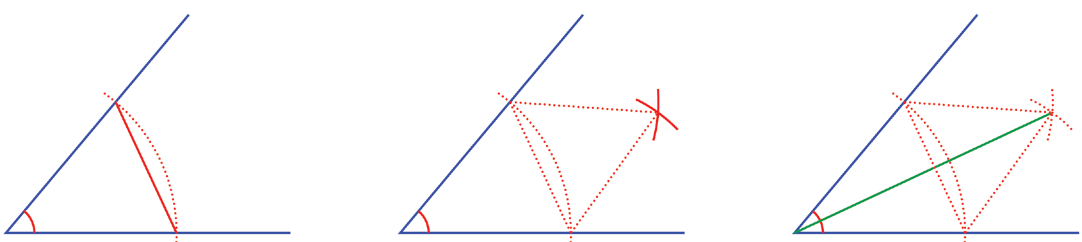
A line which divides an angle into two equal parts is called the **bisector of the angle**:



How do we draw the bisector of a given angle?

We have seen that in an isosceles triangle, the perpendicular from the vertex joining the equal sides to the opposite side, bisects the angle at this vertex.

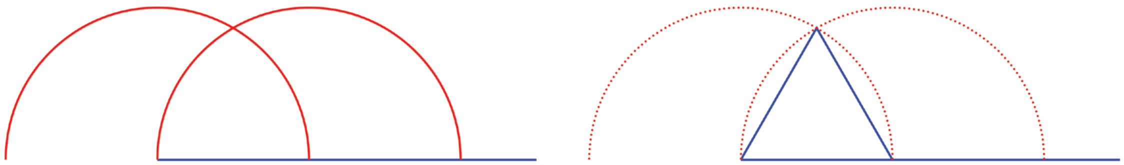
If we mark points on the sides of the angle, at the same distance from the vertex, we get an isosceles triangle; and the perpendicular from the vertex of the angle to the base of the isosceles triangle gives the bisector of the angle:



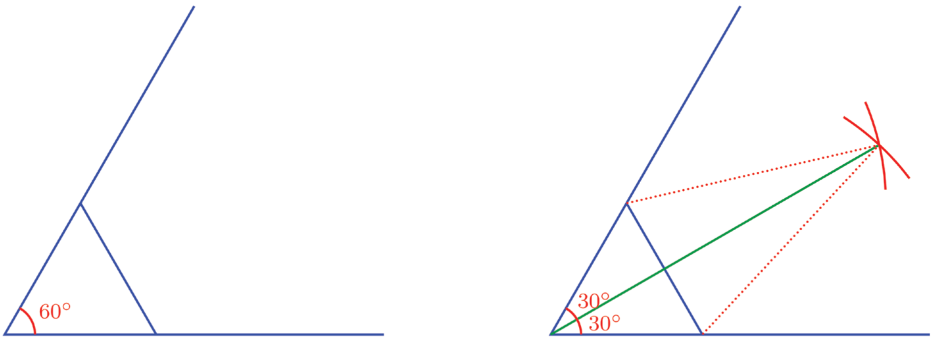
We can use this to draw a 30° angle using only ruler and compass.

First we draw a 60° angle.

For that we need only draw a line and draw an equilateral triangle with one of its ends as a vertex (the section, **Star picture** of the lesson **Triangles** in the class 7 textbook).



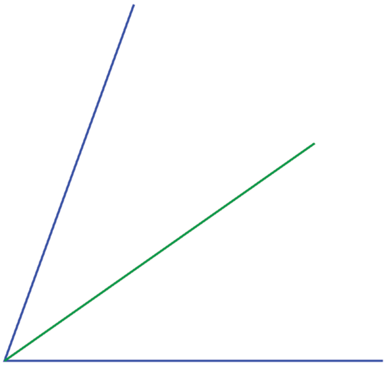
Since each angle of an equilateral triangle is 60°, we have such an angle at one end of the line drawn first. Drawing its bisector, we get a 30° angle:



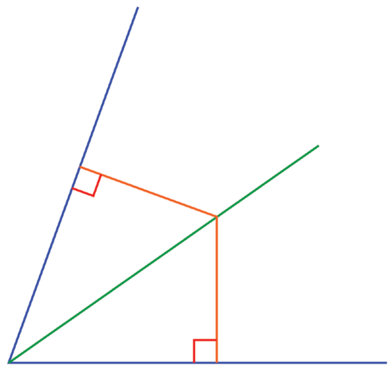
We have seen that any point on the perpendicular bisector of a line is at the same distance from the end points of that line; and on the other hand, any point which is at the same distance from the endpoints of a line is on the perpendicular bisector of that line.

Angle bisectors also have a similar property.

To see this, draw an angle and its bisector.

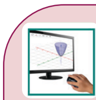


From some point on the bisector, draw perpendiculars to the sides of the angle:



In the second picture we have two right triangles, one on top and other at the bottom. Since the green line is the bisector of the angle, one angle of a triangle is equal to one angle of the other. Since both are right triangles, all three angles of one triangle are equal to the angles of the other.

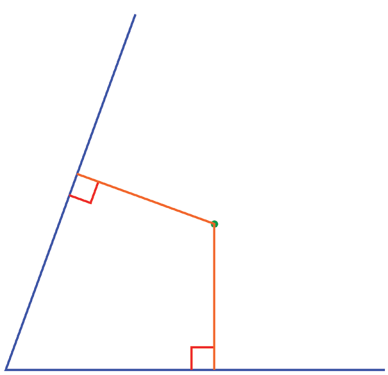
The triangles have the same hypotenuse and the angles at its ends are also the same. So, the red sides of the triangles are of equal length. This means, the perpendicular distances from any point on the angle bisector to the sides are equal.

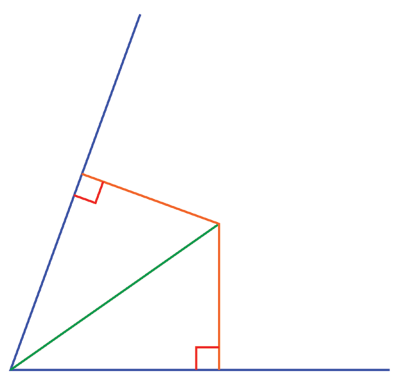


Draw an angle *ABC* in GeoGebra. Select the Angle Bisector tool and click on *A, B, C* to draw the bisector of the angle at *B*. Mark a point D on this bisector and draw perpendiculars from this point to *AB* and *BC*. Mark the points E and F where these perpendiculars meet *AB* and *BC*. Draw the lines *DE* and *DF* and mark their lengths. Move *D* along the bisector and check these lengths.

On the other hand, if the perpendicular distances from a point to the sides of an angle are equal, does this point lie on the bisector of that angle?

Imagine such a point and the line joining it to the vertex of the angle:





We get two right triangles with the same hypotenuse as before; and their red sides are equal. So, by Pythagoras' Theorem, their third sides are also equal. Since the sides of the two triangles are of the same length, their angles are also equal. The angles at the left vertex of these triangles are just the parts into which the green line divides the original angle. Since these are equal, the green line is the bisector of this angle.

So, what can we say about the bisector of an angle?

Any point on the bisector of an angle is at the same perpendicular distance from the sides of the angle; on the other hand, any point at the same perpendicular distance from two sides of an angle lies on the bisector of that angle

(1) Draw the angles below:

(i) $37\frac{1}{2}^\circ$ (ii) $62\frac{1}{2}^\circ$

(2) Use only ruler and compass to draw the following:

(i) The angles below

(a) 45° (b) 135° (c) 75° (d) 15°

(ii) The triangle with one side of length 6 centimetres and angles $67\frac{1}{2}^\circ$ and $22\frac{1}{2}^\circ$ at its ends.

Notes

Notes

Notes

CONSTITUTION OF INDIA

Part IV A

FUNDAMENTAL DUTIES OF CITIZENS

ARTICLE 51 A

Fundamental Duties- It shall be the duty of every citizen of India:

(a) to abide by the Constitution and respect its ideals and institutions, the National Flag and the National Anthem;

(b) to cherish and follow the noble ideals which inspired our national struggle for freedom;

(c) to uphold and protect the sovereignty, unity and integrity of India;

(d) to defend the country and render national service when called upon to do so;

(e) to promote harmony and the spirit of common brotherhood amongst all the people of India transcending religious, linguistic and regional or sectional diversities; to renounce practices derogatory to the dignity of women;

(f) to value and preserve the rich heritage of our composite culture;

(g) to protect and improve the natural environment including forests, lakes, rivers, wild life and to have compassion for living creatures;

(h) to develop the scientific temper, humanism and the spirit of inquiry and reform;

(i) to safeguard public property and to abjure violence;

(j) to strive towards excellence in all spheres of individual and collective activity so that the nation constantly rises to higher levels of endeavour and achievements;

(k) who is a parent or guardian to provide opportunities for education to his child or, as the case may be, ward between age of six and fourteen years.

CHILDREN'S RIGHTS

Dear Children,

Wouldn't you like to know about your rights? Awareness about your rights will inspire and motivate you to ensure your protection and participation, thereby making social justice a reality. You may know that a commission for child rights is functioning in our state called the ***Kerala State Commission for Protection of Child Rights.***

Let's see what your rights are:

- Right to freedom of speech and expression.
- Right to life and liberty.
- Right to maximum survival and development.
- Right to be respected and accepted regardless of caste, creed and colour.
- Right to protection and care against physical, mental and sexual abuse.
- Right to participation.
- Protection from child labour and hazardous work.
- Protection against child marriage.
- Right to know one's culture and live accordingly.
- Protection against neglect.
- Right to free and compulsory education.
- Right to learn, rest and leisure.
- Right to parental and societal care, and protection.

Major Responsibilities

- Protect school and public facilities.
- Observe punctuality in learning and activities of the school.
- Accept and respect school authorities, teachers, parents and fellow students.
- Readiness to accept and respect others regardless of caste, creed or colour.

Contact Address:

Kerala State Commission for Protection of Child Rights
'Sree Ganesh', T. C. 14/2036, Vanross Junction
Kerala University P. O., Thiruvananthapuram - 34, Phone : 0471 - 2326603
Email: childrights.cpcr@kerala.gov.in, rte.cpcr@kerala.gov.in
Website : www.kescpcr.kerala.gov.in

Child Helpline - 1098, Crime Stopper - 1090, Nirbhaya - 1800 425 1400

Kerala Police Helpline - 0471 - 3243000/44000/45000

Online ***R. T. E Monitoring : www.nireekshana.org.in***