MATHEMATICS

Part - 2

Standard

VII

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State Council of Educational Research and Training (SCERT) Kerala

2024

THE NATIONAL ANTHEM

Jana-gana-mana adhinayaka, jaya he
Bharatha-bhagya-vidhata
Punjab-Sindh-Gujarat-Maratha
Dravida-Utkala-Banga
Vindhya-Himachala-Yamuna-Ganga
Uchchala-Jaladhi-taranga
Tava subha name jage,
Tava subha asisa mage,
Gahe tava jaya gatha
Jana-gana-mangala-dayaka jaya he
Bharatha-bhagya-vidhata
Jaya he, jaya he, jaya he,
Jaya jaya jaya, jaya he.

PLEDGE

India is my country. All Indians are my brothers and sisters.

I love my country, and I am proud of its rich and varied heritage. I shall always strive to be worthy of it.

I shall give my parents, teachers and all elders, respect and treat everyone with courtesy.

To my country and my people, I pledge my devotion. In their well-being and prosperity alone, lies my happiness.

MATHEMATICS

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Dear children,

We've acquired some of the basic concepts of mathematics.
Such as counting numbers, fractions, decimal forms and some algebra too. We've also had ample opportunities to use these to solve problems logically, explain them in terms of cause and effect, and to complete patterns.
We are moving ahead. To more applications of the concepts learnt, to more computational techniques, to develop more skill in recognising relations between numbers, explaining them in our own language and in the language of mathematics, to more deeper analysis of geometry, to more complex problem solving, to more possibilities of mathematical and computational thinking.

Let's march ahead together with confidence, thinking, enquiring and enjoying ourselves.

With love and regards,

Dr. Jayaprakash R.K.
Director
SCERT, Kerala

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Certain icons are used in this textbook for convenience


Let's do problems


ICT possibilities


Project

THE CONSTITUTION OF INDIA

PREAMBLE

WE, THE PEOPLE OF INDIA, having solemnly resolved to constitute India into a [1]**[SOVEREIGN SOCIALIST SECULAR DEMOCRATIC REPUBLIC]** and to secure to all its citizens :

JUSTICE, social, economic and political;

LIBERTY of thought, expression, belief, faith and worship;

EQUALITY of status and of opportunity; and to promote among them all

FRATERNITY assuring the dignity of the individual and the [2][unity and integrity of the Nation];

IN OUR CONSTITUENT ASSEMBLY this twenty-sixth day of November, 1949 do **HEREBY ADOPT, ENACT AND GIVE TO OURSELVES THIS CONSTITUTION.**

---

1. Subs. by the Constitution (Forty-second Amendment) Act, 1976, Sec.2, for "Sovereign Democratic Republic" (w.e.f. 3.1.1977)
2. Subs. by the Constitution (Forty-second Amendment) Act, 1976, Sec.2, for "Unity of the Nation" (w.e.f. 3.1.1977)

8 Repeated Multiplication

Factors

We have seen how numbers can be split into products of prime numbers.

How do we do it for 128?

$$128 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$$

It is not easily seen how many 2's are there in this; also it is tedious to write it out.

So, we write this product in a shortened form as 2^7 (read, "two to the seventh power"); that is, a shorthand notation for seven two's multiplied together

$$2^7 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$$

How do we write 243 like this?

$$243 = 3 \times 3 \times 3 \times 3 \times 3 = 3^5$$

Thus we write repeated multiplication of the same number in this form, just as we write repeated addition as multiplication.

For example,

$5 + 5 = 2 \times 5$	$5 \times 5 = 5^2$
$5 + 5 + 5 = 3 \times 5$	$5 \times 5 \times 5 = 5^3$
$5 + 5 + 5 + 5 = 4 \times 5$	$5 \times 5 \times 5 \times 5 = 5^4$

And we can continue this.

The operation of multiplying a number by itself repeatedly is called *exponentiation*. The number showing how many are multiplied together is called the *exponent*.

We write the exponent in a smaller size, to the right and slightly above the number multiplied.

The numbers got by repeatedly multiplying a number by itself are called *powers*. For example,

$2 \times 2 \times 2$	$2^3 = 8$	Third power of two
3×3	$3^2 = 9$	Second power of three
$7 \times 7 \times 7 \times 7$	$7^4 = 2401$	Fourth power of seven

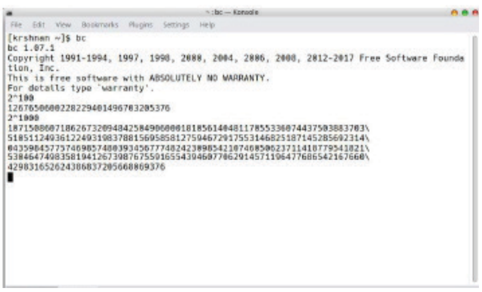
We can consider any number as the first power of itself.

Machine math

Exponentiation is an operation using two numbers, just like addition, subtraction, multiplication and division. But unlike the other operations, exponentiation is not written with an operation symbol between the numbers.

In calculators and computers, we cannot enter exponents by reducing the size and lifting up. Usually the symbol ^ is used to denote exponentiation in such cases. For example, we type 2^5 to get 2^5

Most Linux systems include a calculator named **bc**, which can handle very large numbers. It can be accessed by typing bc in the terminal. See how it calculates 2^{100} and 2^{1000}:



Let's return to factorization. We wrote 128 as a power of 2 and 243 as a power of 3:

$$128 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^7$$
$$243 = 3 \times 3 \times 3 \times 3 \times 3 = 3^5$$

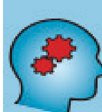
How do we split 576 as a product of primes?

$$576 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3$$
$$= 2^6 \times 3^2$$

We know that any natural number greater than 1 can be written as a product of primes.

Now we can modify this a little:

> Any natural number greater than 1 can be written either as the power of a prime or as the product of powers of different primes



Write each number below either as a power of a single prime or as a product of powers of different primes:

(i) 125 (ii) 72 (iii) 100

(iv) 250 (v) 3600 (vi) 10800

Powers of fractions

What is the area of a square of sides 4 metres?

What if the length of the sides is $\frac{1}{4}$ metre?

Just as we write 4×4 as 4^2 we can write

$$\frac{1}{4} \times \frac{1}{4} = \left(\frac{1}{4}\right)^2$$

Now according to the definition of fractions

$$\frac{1}{4} \times \frac{1}{4} = \frac{1}{4 \times 4}$$

and so we get

$$\left(\frac{1}{4}\right)^2 = \frac{1}{4^2}$$

Thus, using algebra, we can say this in general:

The area of a square with sides of length x is x^2

(x^2 looks a lot nicer than $x \times x$ or xx, doesn't it?)

Now what if we use decimal forms?

The area of a square of sides 0.25 metres is

$(0.25)^2 = 0.25 \times 0.25 = 0.0625$ square metre

In the same way, the volume of a cube with lengths of edges $\frac{1}{2}$ metre can be written in shorthand:

$$\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \left(\frac{1}{2}\right)^3 \text{ cubic metre}$$

(the shortening is only in notation; we must do the actual multiplication to get it as $\frac{1}{8}$).

Scientific notation

Numbers, especially large numbers, are written in a special way in most of the scientific articles: as the product of a number between 1 and 10 by a power of 10. For example,

2376	2.376×10^3
237.6	2.376×10^2
23.76	2.376×10

It is called *scientific notation*

And in decimal form?

The volume of a cube with lengths of edges 0.5 metre is

$$(0.5)^3 = 0.5 \times 0.5 \times 0.5$$
$$= 0.125 \text{ cubic metre}$$

Using algebra, we can write it as follows:

The volume of a cube with the length of the edges x is x^3

So then, what is the volume of a cube of edges $2\frac{1}{4}$ metres?

$$\left(2\frac{1}{4}\right)^3 = \left(\frac{9}{4}\right)^3 = \frac{9}{4} \times \frac{9}{4} \times \frac{9}{4} = \frac{9 \times 9 \times 9}{4 \times 4 \times 4} = \frac{729}{64}$$

So, volume is $\frac{729}{64}$ cubic metres

We can split this into quotient and remainder and write

$$\frac{729}{64} = 11\frac{25}{64}$$

or use a calculator to compute

$$\frac{729}{64} = 11.390625$$

We can convert this to cubic centimetres, if we want:

$$1 \text{ metre} = 100 \text{ centimetres}$$
$$= 10^2 \text{ centimetres}$$
$$1 \text{ cubic metre} = 100 \times 100 \times 100 \text{ cubic centimetres}$$
$$= 10^6 \text{ cubic centimetres}$$

So,

11.390625 cubic metres

$$= 11.390625 \times 10^6 \text{ cubic centimetres}$$

$$= 11390625 \text{ cubic centimetres}$$

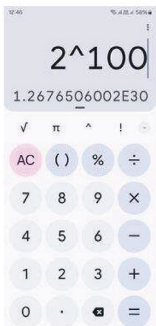
Now let's look at some other problems.

We can easily compute the powers of 2, one after the other:

$$2^2 = 2 \times 2 = 4$$
$$2^3 = 4 \times 2 = 8$$
$$2^4 = 8 \times 2 = 16$$
$$2^5 = 16 \times 2 = 32$$

Machine limits

Calculators use the scientific notation to display large numbers:



In this, 1.267650600E30 stands for $1.267650600 \times 10^{30}$, which means 1267650600 followed by 21 zeros.

And the answer is not exact, just an approximation of 2^{100}. All computing machine have limits on the size of the numbers they can handle. The calculator shown here can handle only up to twelve-digit numbers. Since the calculator **bc** discussed earlier has very high limits, it can compute 2^{100} or even 2^{1000} exactly.

Using these, we can also easily compute powers of $\frac{1}{2}$

$$\left(\frac{1}{2}\right)^2 = \frac{1}{2^2} = \frac{1}{4}$$

$$\left(\frac{1}{2}\right)^3 = \frac{1}{2^3} = \frac{1}{8}$$

$$\left(\frac{1}{2}\right)^4 = \frac{1}{2^4} = \frac{1}{16}$$

$$\left(\frac{1}{2}\right)^5 = \frac{1}{2^5} = \frac{1}{32}$$

We note that the powers of 2 keep on increasing, while the powers of $\frac{1}{2}$ keep on decreasing.

Why is this so?

Each multiplication by 2 doubles, while each multiplication by $\frac{1}{2}$ halves.

We can say this in general:

> Powers of numbers greater than 1 steadily increase. Powers of numbers greater than 0 and less than 1 steadily decrease. All powers of 1 remain 1 and all powers of 0 remain 0

Now try these problems:

(1) Calculate the powers below as fractions:

(i) $\left(\frac{2}{3}\right)^2$ (ii) $\left(1\frac{1}{2}\right)^2$ (iii) $\left(\frac{2}{5}\right)^3$ (iv) $\left(2\frac{1}{2}\right)^3$

(2) Calculate the powers below in decimal form:

(i) $(0.5)^2$ (ii) $(1.5)^2$ (iii) $(0.1)^3$ (iv) $(0.01)^3$

(3) Using $15^3 = 3375$ calculate the powers below:

(i) $(1.5)^3$ (ii) $(0.15)^3$ (iii) $(0.015)^3$

Power of powers

There is an old tale which shows how quickly the powers of a number increase. A king was defeated in a chess game and he asked the winner what gift he wanted. He wanted one grain of rice in the first square of the chessboard, two grains on the second square, four on the third and so on, doubling the number for each square, till all the squares in the board were filled. The king thought this could be done easily, but even after all the granaries of the country were exhausted, not even half the board could be filled.

Let's look at the math behind this. What is needed here is the sum of the powers of 2 from 1 to 63. Using a calculator such as **bc**, this can be calculated and it turns out to be larger than ten quadrillion (10^{16}). If the weight of one grain of rice is taken as 30 milligrams, the total weight of this many grains would be more than five hundred billion tonnes. And note that the total weight of rice produced in the whole world in a year is about five billion tonnes.

Products of powers

We know that 3 times 2 and 5 times 2 added together gives 8 times 2. Why is this so?

$$3 \text{ times } 2 = 2 + 2 + 2$$

$$5 \text{ times } 2 = 2 + 2 + 2 + 2 + 2$$

Adding these gives

$$\underbrace{(2+2+2)}_{3 \text{ times}} + \underbrace{(2+2+2+2+2)}_{5 \text{ times}} = \underbrace{2+2+2+2+2+2+2+2}_{8 \text{ times}}$$

We can write this as

$$(3 \times 2) + (5 \times 2) = (3 + 5) \times 2 = 8 \times 2$$

What about powers of 2?

$$3^{rd} \text{ power of } 2 = 2 \times 2 \times 2$$

$$5^{th} \text{ power of } 2 = 2 \times 2 \times 2 \times 2 \times 2$$

Since these are all products, let's find the product of these two, instead of their sum:

$$\underbrace{(2 \times 2 \times 2)}_{3 \text{ times}} \times \underbrace{(2 \times 2 \times 2 \times 2 \times 2)}_{5 \text{ times}} = \underbrace{2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2}_{8 \text{ times}}$$

How do we write this using exponents?

$$2^3 \times 2^5 = 2^{3+5} = 2^8$$

How about writing $3^2 \times 3^4$ like this?

$$3^2 = 3 \times 3$$

$$3^4 = 3 \times 3 \times 3 \times 3$$

Multiplying

$$3^2 \times 3^4 = \underbrace{(3 \times 3)}_{2 \text{ times}} \times \underbrace{(3 \times 3 \times 3 \times 3)}_{4 \text{ times}} = \underbrace{3 \times 3 \times 3 \times 3 \times 3 \times 3}_{6 \text{ times}}$$

Thus

$$3^2 \times 3^4 = 3^6$$

So what can we say in general about the product of two powers of a number?

In multiplying two powers of a number, the exponents should be added

Using algebra, we can write it as follows:

$x^m \times x^n = x^{m+n}$ for all numbers x and all natural numbers m and n

We must note two things here:

(i) The product of two powers of the *same number* is a power of that number

(ii) The exponent of the product is the *sum* of the exponents of the numbers multiplied

We can extend this to products of more than two powers.

For example, we can compute $3^2 \times 3^5 \times 3^4$ like this:

$$\begin{aligned} 3^2 \times 3^5 \times 3^4 &= (3^2 \times 3^5) \times 3^4 \\ &= 3^7 \times 3^4 \\ &= 3^{11} \end{aligned}$$

We can use this to find the prime factorization of a product.

For example consider the product 96 × 144. We first write each number as a product of powers of different primes:

$$\begin{aligned} 96 &= 2 \times 2 \times 2 \times 2 \times 2 \times 3 = 2^5 \times 3 \\ 144 &= 2 \times 2 \times 2 \times 2 \times 3 \times 3 = 2^4 \times 3^2 \end{aligned}$$

and then compute the product as

$$\begin{aligned} 96 \times 144 &= (2^5 \times 3) \times (2^4 \times 3^2) \\ &= (2^5 \times 2^4) \times (3 \times 3^2) \\ &= 2^9 \times 3^3 \end{aligned}$$

Let's look at another example:

Find the prime factorization of the product of the numbers from 1 to 9.

We need not do anything with the prime numbers 2, 3, 5, 7 among these. The others we split into prime factors:

$$4 = 2^2$$
$$6 = 2 \times 3$$
$$8 = 2^3$$
$$9 = 3^2$$

So,

$$\begin{aligned}1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 \times 9 &= 2 \times 3 \times 2^2 \times 5 \times (2 \times 3) \times 7 \times 2^3 \times 3^2 \\ &= (2 \times 2^2 \times 2 \times 2^3) \times (3 \times 3 \times 3^2) \times 5 \times 7 \\ &= 2^7 \times 3^4 \times 5 \times 7\end{aligned}$$

Note that in these problems we found the prime factorization of products, without actually computing the products

Now try these problems:

(1) Write each product below as the product of powers of different primes:

(i) 72 × 162 (ii) 225 × 135 (iii) 105 × 175 (iv) 25 × 45 × 75

(2) Write the product of the numbers from 1 to 15 as the product of powers of different primes

(3) Consider the numbers from 1 to 25

(i) Which of them are divisible by 2, but not by 4?

(ii) Which of them are divisible by 4, but not by 8?

(iii) Which of them are divisible by 8, but not by 16?

(iv) Which of them are divisible by 16?

(v) What is the highest power of 2 that divides the product of the numbers from 1 to 25 without remainder?

(4) Consider the product of the numbers from 1 to 25

(i) What is the highest power of 5 which divides this product without remainder?

(ii) And the highest power of 10 dividing this product without remainder?

(iii) How many zeros does this product end with?

Quotient of powers

What is 256 ÷ 32 ?

It is convenient to factorize before dividing:

$$256 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^8$$

$$32 = 2 \times 2 \times 2 \times 2 \times 2 = 2^5$$

So what we have to find is

$$2^8 \div 2^5$$

The question, in other words, is by what number 2^5 is to be multiplied to get 2^8. We have seen earlier that

$$2^5 \times 2^3 = 2^8$$

Thus

$$2^8 \div 2^5 = 2^3$$

$$256 \div 32 = 8$$

Let's now try dividing 15625 by 625.

Dividing repeatedly by 5, we can write each as a power of 5:

$$15625 = 5^6$$

$$625 = 5^4$$

So, the problem becomes

$$15625 \div 625 = 5^6 \div 5^4$$

In other words, the question becomes this:

By what number should 5^4 be multiplied to get 5^6?

When powers are multiplied, the exponents get added, right?

Here the first power 4 is to be made the second power 6.

So the question becomes this:

What number should be added to 4 to make it 6?

Now putting these thoughts in reverse, we have the answer to the original question:

- $4 + 2 = 6$
- $5^4 \times 5^2 = 5^6$
- $5^6 \div 5^4 = 5^2$
- $15625 \div 625 = 25$

Like this, how do we think about the problem of finding $8^{15} \div 8^6$?

- What is 8^{15} divided by 8^6?
- By what power of 8 should 8^6 be multiplied to get 8^{15}?
- What number should be added to 6 to get 15 ?

Once we find the answer to the last question above, we can reverse our thoughts to find the answer to the first question:

- $15 - 6 = 9$
- $6 + 9 = 15$
- $8^6 \times 8^9 = 8^{15}$
- $8^{15} \div 8^6 = 8^9$

Let's look at the above three problems again:

$$2^8 \div 2^5 = 2^3 = 2^{8-5}$$

$$5^6 \div 5^4 = 5^2 = 5^{6-4}$$

$$8^{15} \div 8^6 = 8^9 = 8^{15-6}$$

Now we can state this as a general principle:

In dividing the larger power of a non-zero number by a smaller power of the same number, the exponents should be subtracted

And we can state this using algebra like this:

$\frac{x^m}{x^n} = x^{m-n}$, for all numbers $x \neq 0$ and for all natural numbers $m > n$

Here we wrote $\frac{x^m}{x^n}$ instead of $x^m \div x^n$, since division is usually written in the fractional form in algebra.

Let's look at another problem:

$$288 \div 36$$

As done earlier, we first factorize the numbers:

$$288 = 2^5 \times 3^2$$

$$36 = 2^2 \times 3^2$$

What next?

We know that in division, we can remove common factors. So,

$$(2^5 \times 3^2) \div (2^2 \times 3^2) = 2^5 \div 2^2$$

Now we can do the division easily:

$$2^5 \div 2^2 = 2^{5-2} = 2^3$$

Let's write all these steps together:

$$\begin{aligned} 288 \div 36 &= (2^5 \times 3^2) \div (2^2 \times 3^2) \\ &= 2^5 \div 2^2 \\ &= 2^3 \\ &= 8 \end{aligned}$$

Now try these problems:

1. Calculate the following quotients:

   (i) $512 \div 64$ (ii) $3125 \div 125$ (iii) $243 \div 27$ (iv) $1125 \div 45$

2. (i) Write half of 2^{10} as a power of 2.

   (ii) Write one-third of 3^{12} as a power of 3.

We can also use exponents to simplify certain fractions.

For example, let's take $\frac{64}{512}$

We can factorize the numerator and denominator and write

$$64 = 2^6, \quad 512 = 2^9$$

Then we can do the calculation like this:

$$\frac{64}{512} = \frac{2^6}{2^9}$$

$$= \frac{2^6}{2^6 \times 2^3}$$

$$= \frac{2^6}{2^6} \times \frac{1}{2^3}$$

$$= \frac{1}{2^3} = \frac{1}{8}$$

Like this,

$$\frac{49}{343} = \frac{7^2}{7^3}$$

$$= \frac{7^2}{7^2 \times 7}$$

$$= \frac{7^2}{7^2} \times \frac{1}{7}$$

$$= \frac{1}{7}$$

The operations done in these two examples can be stated as a general principle, using algebra:

$\frac{x^m}{x^n} = \frac{1}{x^{n-m}}$, for all numbers $x \neq 0$ and for all natural numbers m < n

Can't you simplify the fractions below like this?

(i) $\frac{27}{243}$ (ii) $\frac{125}{3125}$ (iii) $\frac{48}{64}$ (iv) $\frac{54}{81}$

Multiples and powers

What do we get by adding 3 times 4 and 3 times 6?

3 times 10, right?

Why is this so?

$$3 \text{ times } 4 = 4 + 4 + 4$$

$$3 \text{ times } 6 = 6 + 6 + 6$$

In adding these two, we can pair each 4 with a 6:

(3 times 4) + (3 times 6) = (4 + 6) + (4 + 6) + (4 + 6) = 10 + 10 + 10 = 3 times 10

We can shorten this as

$$(3 \times 4) + (3 \times 6) = 3 \times (4 + 6)$$

What if we take powers instead of multiples?

$$3^{\text{rd}} \text{ power of } 4 = 4 \times 4 \times 4$$

$$3^{\text{rd}} \text{ power of } 6 = 6 \times 6 \times 6$$

And multiply these instead of adding, pairing each 4 with a 6 as before:

(3^{rd} power of 4) × (3^{rd} power of 6) = (4 × 6) × (4 × 6) × (4 × 6) = 3^{rd} power of (4 × 6)

We can write the 3rd power of (4 × 6) as $(4 \times 6)^3$

Then the equation above becomes

$$4^3 \times 6^3 = (4 \times 6)^3$$

Even if we change the numbers 4, 6 and 3 to some others, we get the same relation as above.

So we can say this in general:

The product of the same powers of two numbers is equal to the same power of the product of these numbers

And using algebra?

$x^n y^n = (xy)^n$ for all numbers x, y and for all natural numbers n

We can use this to simplify some multiplications.

For example, consider 625×16

Once we see that

$$625 = 5^4$$
$$16 = 2^4$$

we can easily compute the product as

$$625 \times 16 = 5^4 \times 2^4 = (5 \times 2)^4 = 10^4 = 10000$$

Let's look at another problem:

What is 5 times 2 times 7?

We can calculate like this:

- 5 times 2 is $5 \times 2 = 10$
- 10 times 7 is $10 \times 7 = 70$

We can also do like this:

- 2 times 7 is $2 \times 7 = 14$
- 5 times 14 is $5 \times 14 = 70$

Let's see why the second computation works

$$2 \text{ times } 7 = 7 + 7$$

$$5 \text{ times } (7 + 7) = (7 + 7) + (7 + 7) + (7 + 7) + (7 + 7) + (7 + 7)$$

$$= 10 \text{ times } 7$$

We can write these computations as

$$5 \times (2 \times 7) = (5 \times 2) \times 7$$

How about powers instead of multiples?

What is the 5th power of the 2nd power of 7?

$$2^{\text{nd}} \text{ power of } 7 = 7 \times 7$$

$$5^{\text{th}} \text{ power of } (7 \times 7) = (7 \times 7) \times (7 \times 7) \times (7 \times 7) \times (7 \times 7) \times (7 \times 7)$$

$$= 10^{\text{th}} \text{ power of } 7$$

We can shorten this using exponents, writing $(7 \times 7)^5$ as $(7^2)^5$. Then the above equation becomes

$$(7^2)^5 = 7^{10}$$

We get such a relation even if we use other numbers. What is this relation?

In computing a power of a power of a number, the exponents should be multiplied

This can be written using algebra as an equation:

$(x^m)^n = x^{mn}$ for all numbers x and for all natural numbers m and n

Using this with the result on product of powers, we can find the prime factorization of a power of a number, whose factorization is known.

For example, once we see that

$$144 = 2^4 \times 3^2$$

we can write

$$\begin{aligned} 144^5 &= (2^4 \times 3^2)^5 \\ &= (2^4)^5 \times (3^2)^5 \\ &= 2^{20} \times 3^{10} \end{aligned}$$

Now try these problems:

(1) Calculate the products below in head:

(i) $5^2 \times 4^2$ (ii) $5^3 \times 6^3$ (iii) $25^3 \times 4^3$ (iv) $125^2 \times 8^2$

(2) Write each number below as a product of powers of different primes:

(i) 15^2 (ii) 30^3 (iii) $12^2 \times 21^2$ (iv) $12^2 \times 21^3$

9 Number Relations

Number of Factors

Haven't you heard of prime numbers?

Numbers greater than 1 which have only 1 and itself as factors.

For example, 2, 3, 5, 7, 11, . . .

Each of them have only two factors.

What about the second power of a prime?

For example, $7^2 = 49$

How many factors does it have?

Just three, namely 1, 7, 49, right?

What about the third power, $7^3 = 343$?

Four factors 1, 7, 49, 343

Now can you say how many factors does 7^4 have, without actually computing this number?

5 factors, 1, 7, 7^2, 7^3, 7^4 isn't it?

In the same way, can you say how many factors 11^5 has, without actually calculating this number?

So what can we say in general?

The number of factors of a power of any prime number is one more than the exponent

We can write this using algebra:

If p is a prime number and n is a natural number, then the number of factors of p^n is $n + 1$

The powers of a prime are got by repeatedly multiplying it by itself. And this result shows how the number of factors change for each such multiplication

Now what about the product of a prime number by another?

For example, let's look at $5 \times 7 = 35$.

First let's take the factors of 5, which are 1 and 5. These are factors of 35 also.

Next if we multiply these two factors by 7, the products 7 and 35 are also factors of 35.

So, how many factors of 35?

1	5	(Factors of 5)
7	35	(Factors of 5 multiplied by 7)

Four factors in all.

What about $5^2 \times 7 = 25 \times 7 = 175$?

First the factors of $5^2 = 25$:

$$1,\ 5,\ 5^2 \text{ that is, } 1,\ 5,\ 25$$

Next three more factors got by multiplying each of these by 7:

1	5	5^2	that is	1	5	25
7	5×7	$5^2 \times 7$		7	35	175

Altogether, $3 + 3 = 3 \times 2 = 6$ factors.

What about $5^2 \times 7^2$?

Let's write down the factors without actually computing the number.

The factors of $5^2 \times 7$ we found earlier are all factors of $5^2 \times 7^2$ also, right?

What are the new factors?

In the table above, we multiplied the factors 1, 5, 5^2 in the first row by 7, to get the factors in the second row.

Now if we multiply the factors in the first row by 7^2, we get the remaining factors of $5^2 \times 7^2$;

$$1 \times 7^2,\ 5 \times 7^2,\ 5^2 \times 7^2$$

Combining all these, we get

1	5	5^2	(factors of 25)
7	5×7	$5^2 \times 7$	(factors of 25 multiplied by 7)
7^2	5×7^2	$5^2 \times 7^2$	(factors of 25 multiplied by 7^2)

Altogether $3 \times 3 = 9$ factors.

Now if we want the factors of $5^2 \times 7^3$ we need write one more row, multiplying by 7^3, the numbers 1, 5, 5^2 in the first row:

1	5	5^2
7	5×7	$5^2 \times 7$
7^2	5×7^2	$5^2 \times 7^2$
7^3	5×7^3	$5^2 \times 7^3$

And the number of factors is $3 \times 4 = 12$

What about $5^3 \times 7^3$?

We have to put 5^3 also in the first row; and below it put the numbers $5^3 \times 7$, $5^3 \times 7^2$ and $5^3 \times 7^3$ as a new column:

1	5	5^2	5^3
7	5×7	$5^2 \times 7$	$5^3 \times 7$
7^2	5×7^2	$5^2 \times 7^2$	$5^3 \times 7^2$
7^3	5×7^3	$5^2 \times 7^3$	$5^3 \times 7^3$

The number of factors becomes $4 \times 4 = 16$.

If we take other numbers instead of 5 and 7, then the actual factors change, but we can compute the number of factors in the same way

For example, how many factors does the number $11^4 \times 17^2$ have?

We can think like this:

- 5 factors of 11^4
- 5 more, got by multiplying each of these by 17
- Another 5, by multiplying each number in the first set by 17^2
- Altogether, $5 \times 3 = 15$ factors

This also gives the tabulation of the factors below:

1	11	11^2	11^3	11^4
17	11×17	$11^2 \times 17$	$11^3 \times 17$	$11^4 \times 17$
17^2	11×17^2	$11^2 \times 17^2$	$11^3 \times 17^2$	$11^4 \times 17^2$

So, what can we say in general about the number of factors of the product of two prime powers?

The number of factors of the product of powers of two prime numbers is got by adding one to each exponent and multiplying these numbers

And using algebra, we can state it like this:

Factor array

If p and q are two different primes and m, n are any two natural numbers, then we can write all the factors of $p^m q^n$ as an array like this:

1	p	p^2	...	p^m
q	pq	p^2q	...	p^mq
q^2	pq^2	p^2q^2	...	p^mq^2
....				
q^n	pq^n	p^2q^n	...	p^mq^n

In the first row, we have $m + 1$ numbers, which are 1 and the m powers of p. And there are n more rows, each got by multiplying the numbers in the first row by the n powers of q. Thus $n + 1$ rows, each row having $m + 1$ numbers. So, altogether $(m + 1)(n + 1)$ numbers

If p and q are two different primes and m, n are any two natural numbers, then the number of factors of $p^m q^n$ is $(m + 1)(n + 1)$.

What if the number has three different prime factors?

For example, how many factors does $5^3 \times 7^2 \times 19$ have?

We know that $5^3 \times 7^2$ has $4 \times 3 = 12$ factors and they can be tabulated like this:

1	5	5^2	5^3
7	5×7	$5^2 \times 7$	$5^3 \times 7$
7^2	5×7^2	$5^2 \times 7^2$	$5^3 \times 7^2$

All these 12 numbers are factors of $5^3 \times 7^2 \times 19$; in addition, the 12 numbers got by multiplying each of these by 19 are also its factors:

19	5×19	$5^2 \times 19$	$5^3 \times 19$
7×19	$5 \times 7 \times 19$	$5^2 \times 7 \times 19$	$5^3 \times 7 \times 19$
$7^2 \times 19$	$5 \times 7^2 \times 19$	$5^2 \times 7^2 \times 19$	$5^3 \times 7^2 \times 19$

Altogether, there are $12 \times 2 = 24$ factors

For $5^3 \times 7^2 \times 19^2$ we will have in addition, another array of 12, got by multiplying the numbers of the first array by 19^2:

19^2	5×19^2	$5^2 \times 19^2$	$5^3 \times 19^2$
7×19^2	$5 \times 7 \times 19^2$	$5^2 \times 7 \times 19^2$	$5^3 \times 7 \times 19^2$
$7^2 \times 19^2$	$5 \times 7^2 \times 19^2$	$5^2 \times 7^2 \times 19^2$	$5^3 \times 7^2 \times 19^2$

And the number of factors becomes $12 \times 3 = 36$.

Thus for each power of a new prime in the number, the number of factors get multiplied by one more than the new exponent.

In general, we can say this:

To compute the number of factors of a number, we write it a product of powers of different prime numbers, and find the product of the numbers got by adding one to each exponent

Now try to do these problems:

(1) Find the number of factors of each number below:

(i) 40 (ii) 54 (iii) 60 (iv) 100 (v) 210

(2) From the number of factors of a number, we can deduce some peculiarities of the number. The table below lists these for number of factors up to 5. Extend it to number of factors up to 10

Number of factors	Form of the number	Algebraic form
1	1	
2	A prime	
3	Second power of a prime	p^2 (p, a prime)
4	Third power of a prime	p^3 (p, a prime)
	Product of two primes	pq (p, q primes)
5	Fourth power of a prime	p^4 (p, a prime)

Common factors

Remember what the common factors of two numbers mean?

For example, if we take 12 and 18, the numbers 2, 3, 6 are factors of both these numbers. In other words, they are common factors of 12 and 18

Do 12 and 18 have any other common factors?

The number 1 is a factor of all numbers, isn't it?

How do we find the common factors of 42 and 70?

We know how to write all the factors of each of these numbers. From these, we can pick out the common factors

There's an easier way. First factorize each into powers of different primes:

$$42 = 2 \times 3 \times 7$$

$$70 = 2 \times 5 \times 7$$

From these, we can see that 2 and 7 are factors of both the numbers 42 and 70.

The remaining factor 3 of 42 is not a factor of 70.

The remaining factor 5 of 70 is not a factor of 42.

So, are 2 and 7, the only common factors?

The product $2 \times 7 = 14$ is also a factor of both, right?

Anything more?

So, the common factors of 42 and 70 are 1, 2, 7, 14

Let's take larger numbers: 504 and 540

First we split them into products of powers of primes:

$$504 = 2^3 \times 3^2 \times 7$$

$$540 = 2^2 \times 3^3 \times 5$$

The prime numbers in both are 2 and 3.

Look at the powers of 2 in each: 2^3 in the first, 2^2 in the second.

Of these, 2^2 is a factor of 2^3; so it is a factor of both 504 and 540

Similarly, looking at the powers of 3, we see that 3^2 is a factor of both these numbers.

Thus the common factors of 504 and 540 are $2^2 \times 3^2$ and its factors. Written out in full they are

1	2	2^2		1	2	4
3	2×3	$2^2 \times 3$	that is	3	6	12
3^2	2×3^2	$2^2 \times 3^2$		9	18	36

Let's look at another example:

Find the common factors of 1260 and 1320

How do we do it?

(i) Write each number as the product of powers of different primes:

$$1260 = 2^2 \times 3^2 \times 5 \times 7$$

$$1320 = 2^3 \times 3 \times 5 \times 11$$

(ii) The primes common to both are 2, 3, 5

(iii) The smaller powers of these primes in the factorizations are 2^2, 3, 5

(iv) The common factors are the factors of $2^2 \times 3 \times 5$

These can be tabulated as before:

1	2	2^2		1	2	4
3	2×3	$2^2 \times 3$	that is	3	6	12
5	2×5	$2^2 \times 5$		5	10	20
3×5	$2 \times 3 \times 5$	$2^2 \times 3 \times 5$		15	30	60

In this, the largest common factor is 60; and all other common factors are factors of 60

Look at the problems done earlier. Are all common factors of the original numbers, factors of the largest common factor?

Now can't you calculate the common factors of any two numbers?

Try these problems:

(1) For each pair of numbers given below, find the largest common factor and all the other common factors:

(i) 45, 75 (ii) 225, 275 (iii) 360, 300 (iv) 210, 504 (v) 336, 588

(2) (i) What is the largest common factor of two different prime numbers?

(ii) Can the largest common factor of two composite numbers be 1?

(iii) If two numbers are divided by their largest common factor, what would be the largest common factor of the quotients?

Project

Just as common factors of two numbers, we can also consider common multiples of two numbers. For example, look at the multiples of 2 and 3:

Multiples of 2 : 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, ...

Multiples of 3 : 3, 6, 9, 12, 15, 18, 21, 24, 27, 30, ...

Which are the numbers which are multiples of both 2 and 3?

Common multiples : 6, 12, 18, ...

Why are all common multiples of 2 and 3, multiples of 6?

On the other hand, are all multiples of 6, also common multiples of 2 and 3?

Think of the following about common multiples:

(1) Are all common multiples of 3 and 4, multiples of a single number?

(2) Are all common multiples of 4 and 6, multiples of 24? If not are they all multiples of some other number?

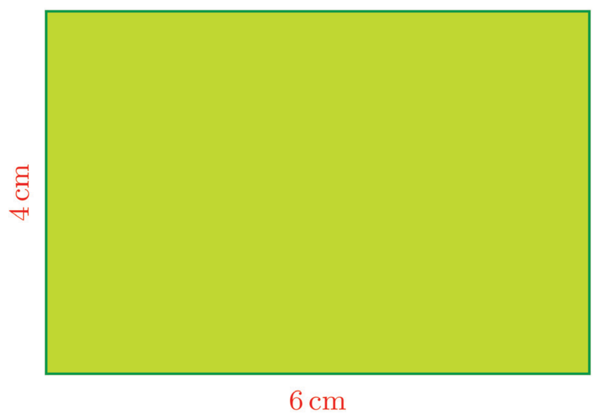
(3) If two numbers are written as the product of powers of different primes, can we find the smallest of their common multiples?

(4) Are all common multiples of two numbers, multiples of the smallest common multiple?

10 AREA OF TRIANGLES

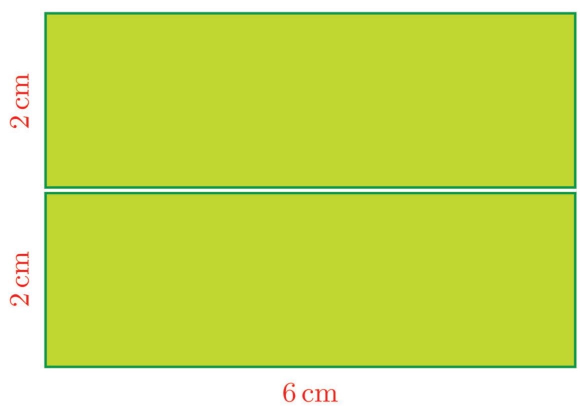
Right triangles

We have learnt in class 5, how we can calculate the area of rectangles. What is the area of the rectangle below?



The area of a rectangle is the product of its width and height, right? So, the area of this rectangle is $6 \times 4 = 24$ square centimetres.

Suppose we cut it horizontally through its middle, to make two smaller rectangles:

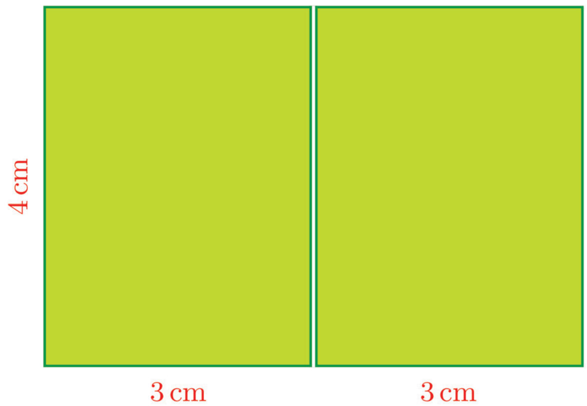


We can compute the area of each small rectangle as $6 \times 2 = 12$ square centimetres.

There's another way to see this.

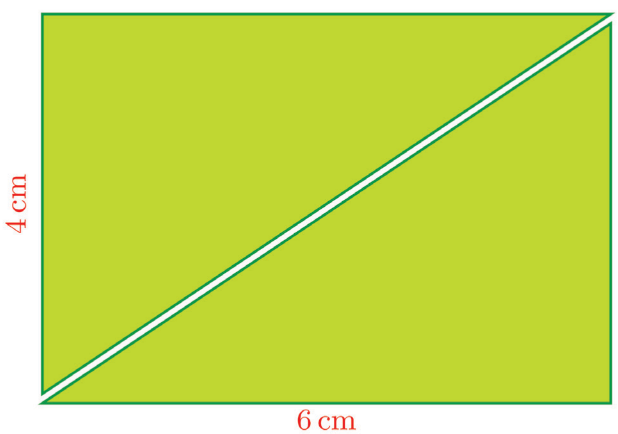
Each small rectangle is half the large rectangle, isn't it? So, the area must also be half. This leads to the calculation of the area of the small rectangles as $24 \times \frac{1}{2} = 12$

What if we cut the large rectangle like this?



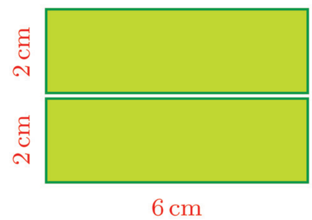
We can calculate the area of a small rectangle by finding the product of its sides as $3 \times 4 = 12$ square centimetres, or just find the half the area of the original rectangle as $24 \times \frac{1}{2} = 12$ square centimetres

Now suppose we cut the rectangle like this:



Area and perimeter

When we cut a rectangle horizontally through its middle, we get two rectangles of half the area:



Is the perimeter of a small rectangle equal to half the perimeter of the large rectangle?

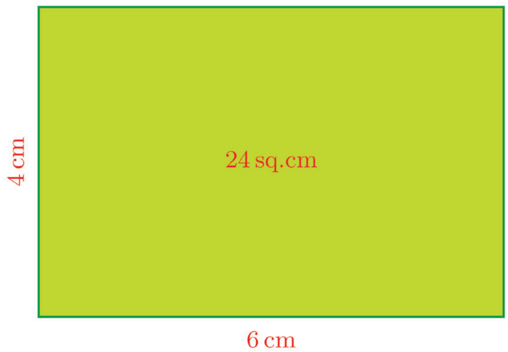
What if we cut like this?

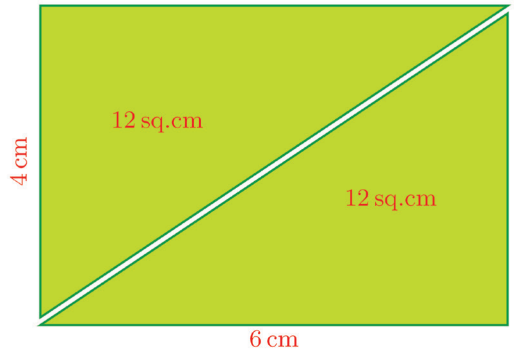
What is the perimeter of each small rectangle? Can we cut out a rectangle of half the perimeter from the large rectangle?

Now the pieces are not rectangles, but triangles; yet each is half the rectangle isn't it?

Cut a rectangle from a thick sheet of paper and cut it along the diagonal like this. The triangles can be placed with one exactly covering the other.

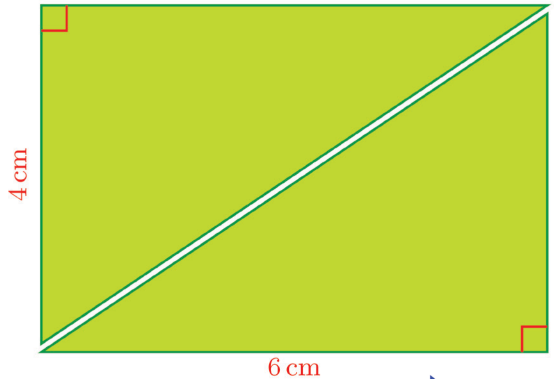
So the area of each triangle is $24 \times \frac{1}{2} = 12$ square centimetres.





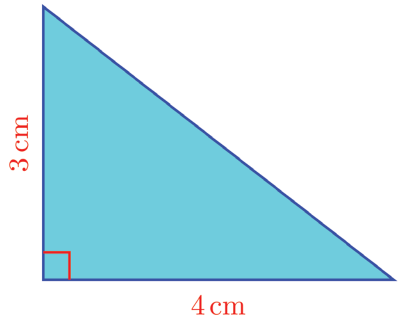
Notice anything special about these triangles?

The adjacent sides of the rectangle are perpendicular to each other; so, in each of these triangles, two of the sides are perpendicular to each other:



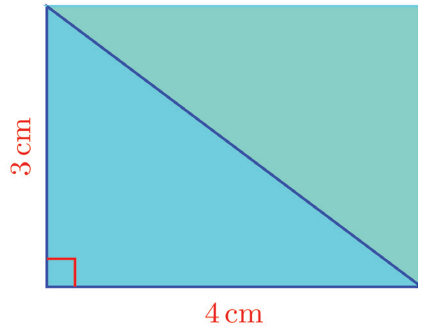
Such a triangle is called a *right triangle*.

See this right triangle:



Can you calculate its area?

We can think of this as half a rectangle:



In other words, it can be made a rectangle by putting a copy of it on top.

The area of the rectangle is $4 \times 3 = 12$ square centimetres; and the triangle is half of the rectangle. So, its area is $12 \times \frac{1}{2} = 6$ square centimetres.

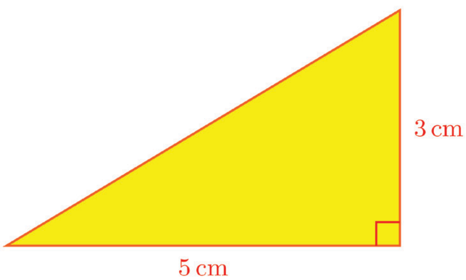
Any right triangle can be seen as half a rectangle like this. How do we compute its area then?

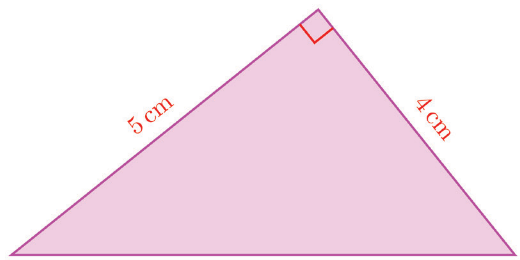
- The width and height of the rectangle are the perpendicular sides of the triangle
- The area of the rectangle is the product of its width and height
- The area of the right triangle is half the area of the rectangle

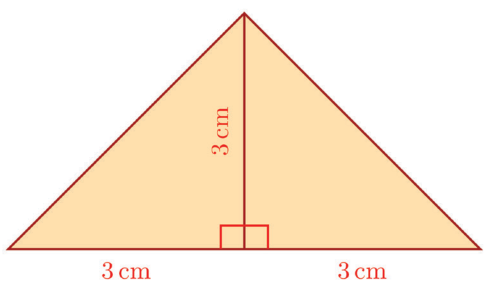
Putting all these together, we can calculate the area of a right triangle without imagining it as half a rectangle every time:

The area of a right triangle is half the product of its perpendicular sides

Now can't you calculate the area of the triangles shown below?

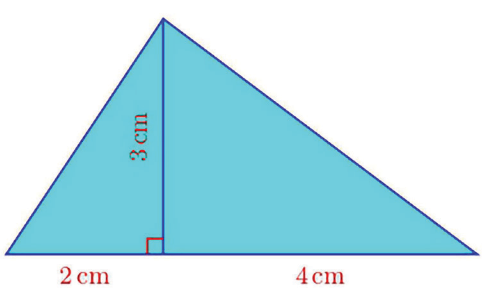




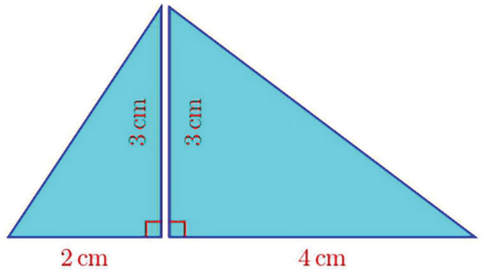


Base and altitude

What is the area of this triangle?



We can consider this triangle as two smaller right triangles joined together:



And we know how to calculate the area of these:

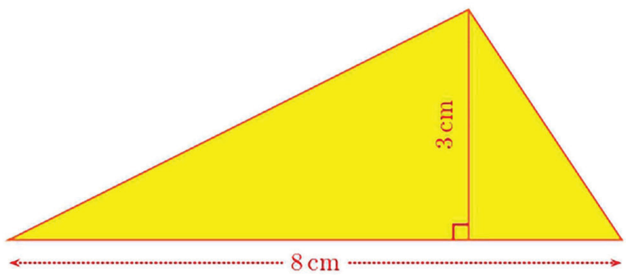
$$\text{Area of the smaller right triangle} = \frac{1}{2} \times 2 \times 3 = 3 \text{ square centimetres}$$

$$\text{Area of the larger right triangle} = \frac{1}{2} \times 4 \times 3 = 6 \text{ square centimetres}$$

Adding these two, we get the area of the original triangle:

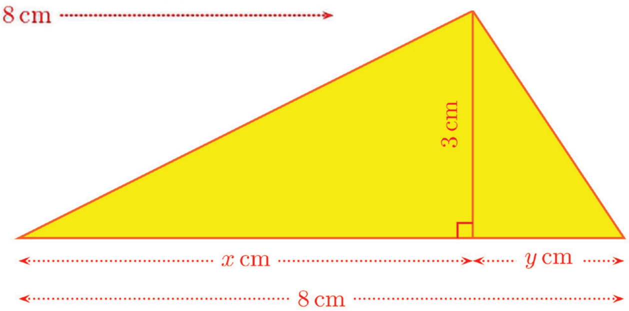
$$\text{Area of the original triangle} = 3 + 6 = 9 \text{ square centimetres}$$

Now how do we calculate the area of this triangle?



Here we don't know the lengths of the bottom sides of the right triangles; so we can't compute the area as before.

Let's take the lengths of these sides as x centimetres and y centimetres:



Then

$$\text{Area of the larger right triangle} = \frac{1}{2} \times x \times 3 = \frac{3}{2}x \text{ square centimetres}$$

$$\text{Area of the smaller right triangle} = \frac{1}{2} \times y \times 3 = \frac{3}{2}y \text{ square centimetres}$$

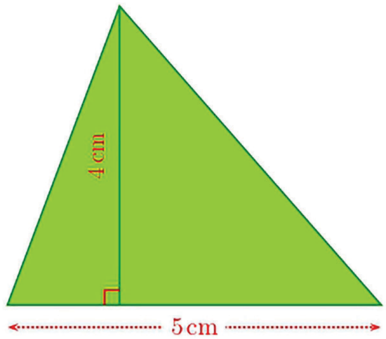
To get the area of the whole triangle we must add these two areas:

$$\frac{3}{2}x + \frac{3}{2}y = \frac{3}{2}(x + y)$$

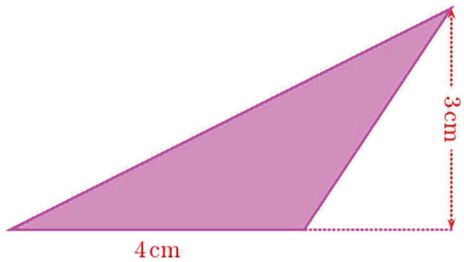
But $x + y = 8$. So the area of the triangle is

$$\frac{3}{2} \times 8 = 12 \text{ square centimetres}$$

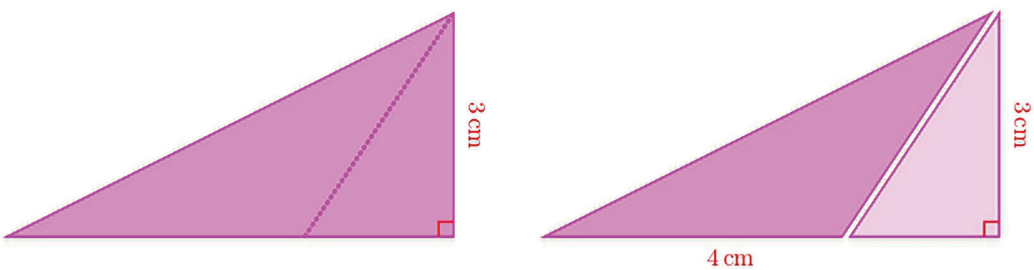
Like this, can you calculate the area of the triangle shown below?



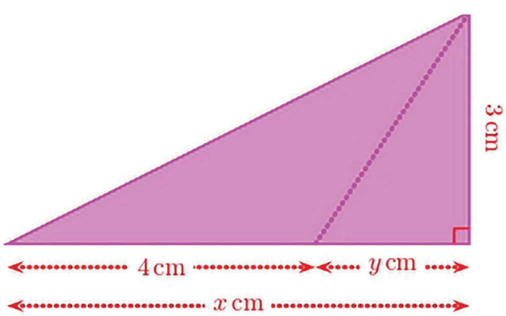
What if the triangle is like this?



In all the problems seen so far, we considered the original triangles as two small right triangles joined together; in this case, we must consider it as a small right triangle cut away from a larger right triangle:



Let's take the length of the bottom side of the larger right triangle as x centimetres and the length of the bottom side of the smaller right triangle as y centimetres:



Then

$$\text{Area of the larger right triangle} = \frac{1}{2} \times x \times 3 = \frac{3}{2}x \text{ square centimetres}$$

$$\text{Area of the smaller right triangle} = \frac{1}{2} \times y \times 3 = \frac{3}{2}y \text{ square centimetres}$$

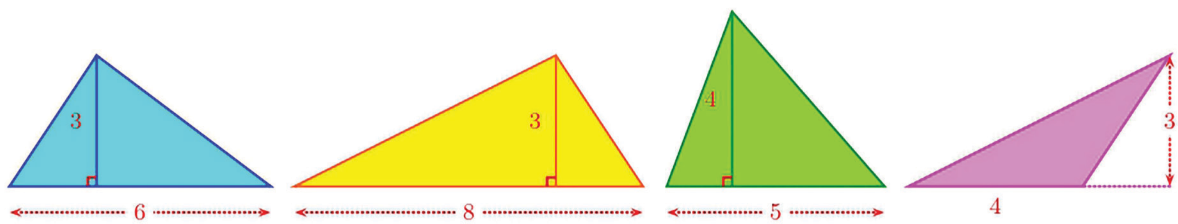
To get the area of the first triangle, we must subtract the area of the smaller right triangle from the area of the larger right triangle:

$$\frac{3}{2}x - \frac{3}{2}y = \frac{3}{2}(x - y)$$

Since $x - y = 4$

we get this as $\frac{3}{2} \times 4 = 6$. Thus the area of the original triangle is 6 square centimetres.

Let's look at all these problems and solutions together:



Area $\frac{1}{2} \times 6 \times 3 = 9$

Area $\frac{1}{2} \times 8 \times 3 = 12$

Area $\frac{1}{2} \times 5 \times 4 = 10$

Area $\frac{1}{2} \times 4 \times 3 = 6$

Changing the lengths does not alter the method of finding areas. So we can make a general statement about computing the area of a triangle:

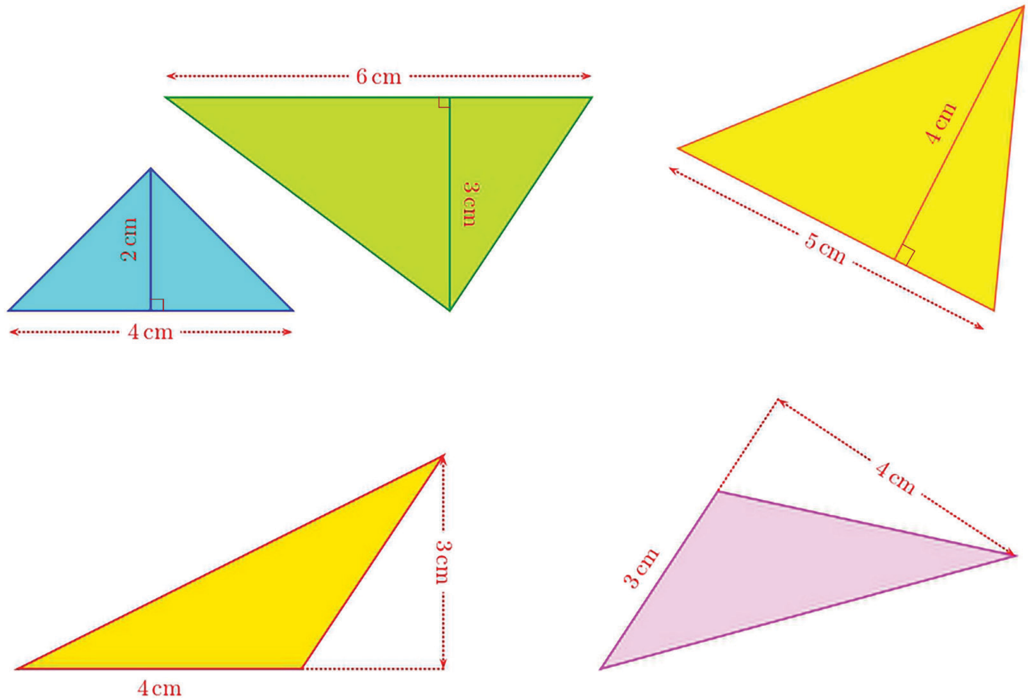
The area of any triangle is half the product of one of the sides and the height from this side to the opposite vertex

If we name the side used to compute the area, the *base* of the triangle and the height to the opposite vertex, the *altitude*, then this can be shortened a bit:

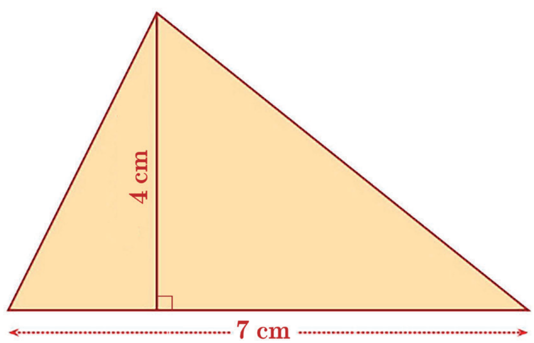
The area of any triangle is half the product of the base and the altitude

Now try these problems:

(1) Find the areas of the triangles shown below:



(2) What is the area of the triangle shown below?



(i) Draw a right triangle of the same area

(ii) Draw a triangle of the same area with one angle greater than a right angle

> Draw a triangle of base 6 and area 12 in GeoGebra. Draw a right triangle and a triangle with two sides equal with the same measures. (It is convenient to use the Grid option for this)

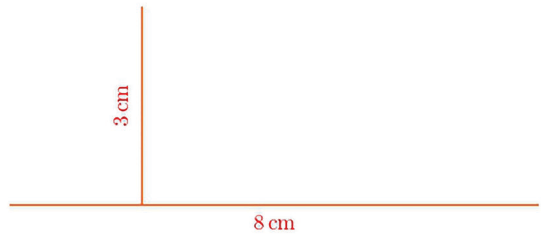
Parallel lines

Can you draw a triangle with one side 8 centimetres and area 12 square centimetres?

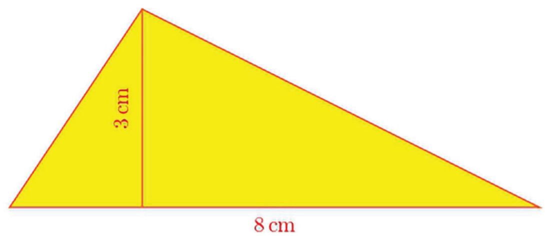
Let's think about it like this:

- Let's take the base as 8 centimetres
  - For the area to be 12 square centimetres, what should be the altitude?
  - The product of base and altitude must be twice the area
- Altitude should be 3 centimetres

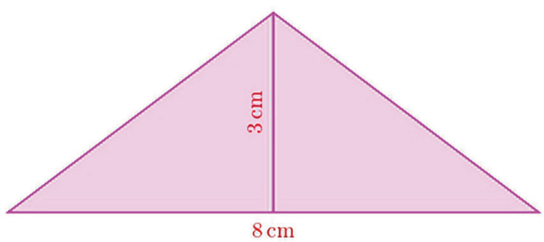
(Remember seeing such a triangle earlier?)



So to draw such a triangle, first draw a line 8 centimetres long and then from some point on it, draw a perpendicular of height 3 centimetres; Joining the top of the perpendicular to the ends of the bottom line makes our triangle, right?

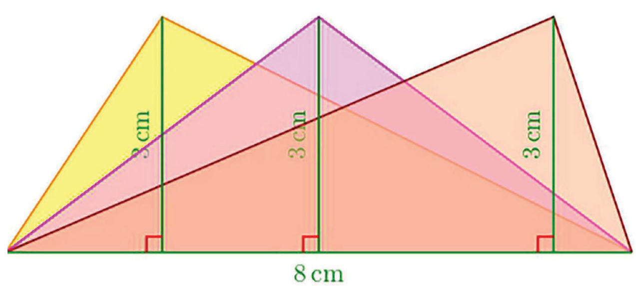


By changing the position of the perpendicular, we can draw another such triangle, can't we?



The lengths of the left and right sides are changed; but since the base and altitude remain the same, the area is not changed.

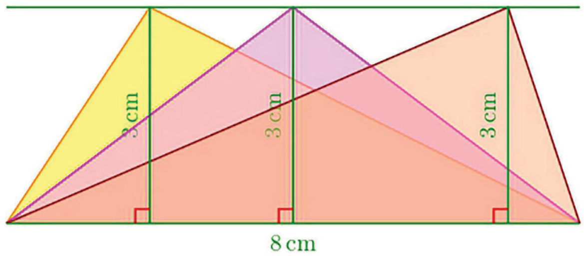
By drawing several perpendiculars, can't we draw many such triangles?



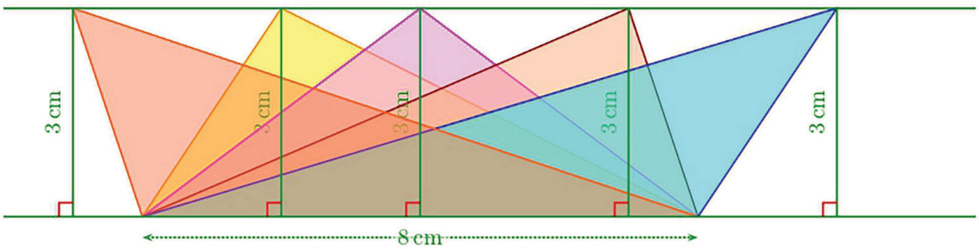
In GeoGebra draw line AB of length 8 and point C at a height 3 above it. Draw a line parallel to AB through C. (Choose the Parallel Line tool; and click on AB and C.) Mark a point D on this line. Use the Polygon tool to draw triangle ABD. By choosing the Area tool and clicking inside the triangle, we get its area. Change the position of D on the line. Does the area change?

By choosing the Distance or Length tool and clicking inside the triangle, we get its perimeter. What is the minimum perimeter when we change the position of D? At what position of D is the perimeter maximum?

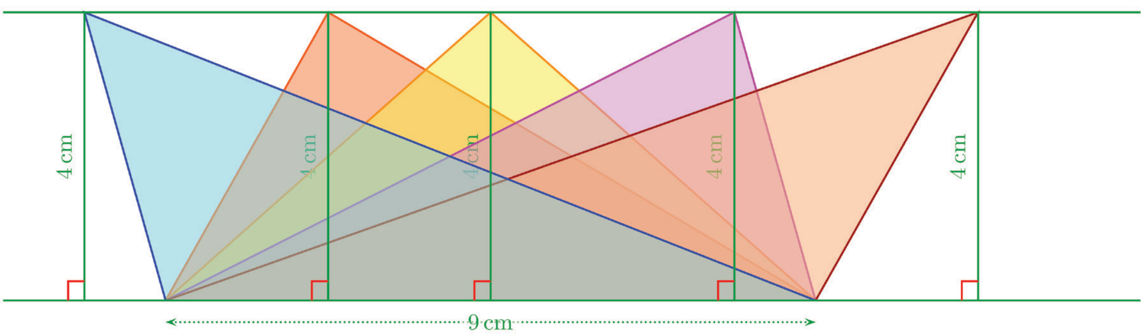
The top vertices of all these triangles are at the same distance from the base, right? So, if we join them, we get a line parallel to the base:



If we extend this line and join any point on it to the endpoints of the bottom line, we get a triangle of base 8 centimetres and altitude 3 centimetres; that is, a triangle of base 8 centimetres and area 12 square centimetres:



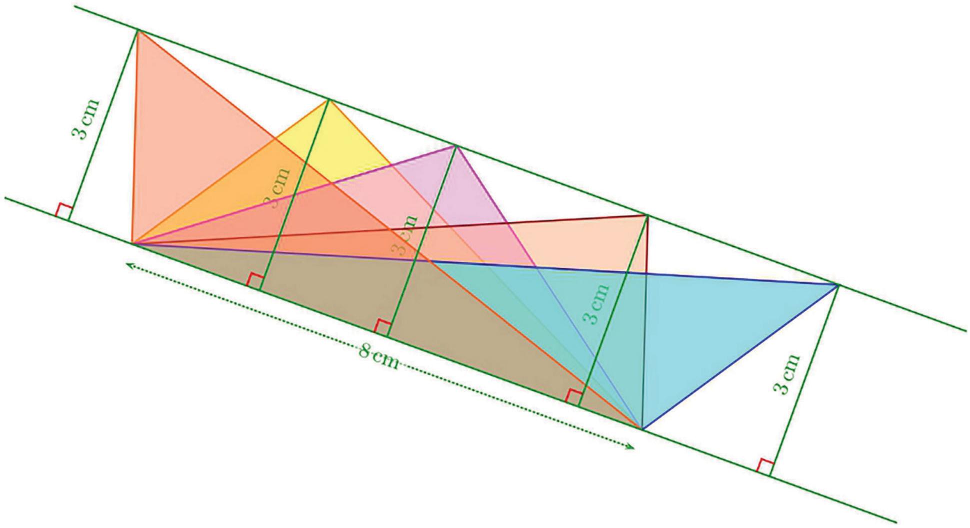
We can draw triangles of other bases and areas in the same way. For example, triangles of base 9 centimetres and area 18 square centimetres can be drawn like this:



This gives us a general result:

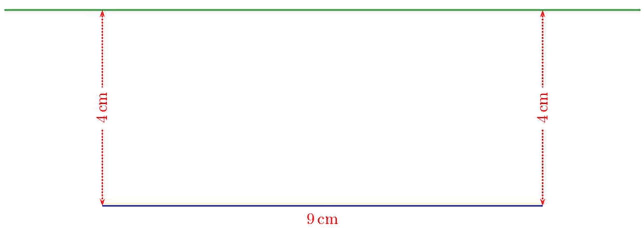
> All triangles with the same base and third vertex on a line parallel to the base, have the same area

In all the pictures above, we draw the base as horizontal. The above result is true even if the base is slanted:



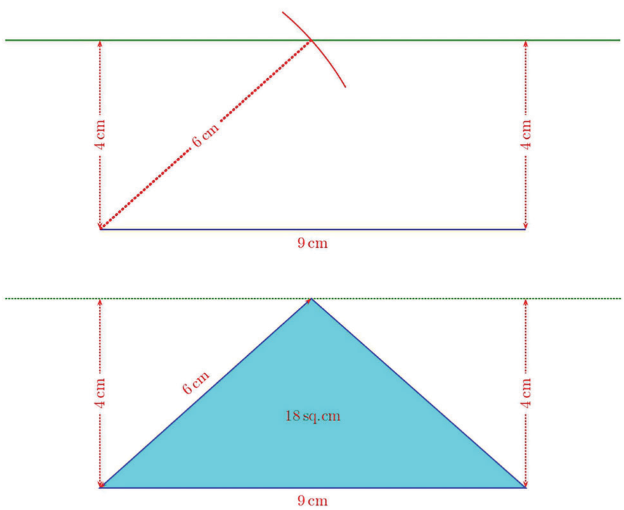
Now let's look at some applications of this result.

We have drawn several triangles with one side 9 centimetres and area 18 square centimetres. What if it is further required that another side should be 6 centimetres?



Look at the picture above. Any point on the top line can be joined to the endpoints of the bottom line to get a triangle of area 18 square centimetres.

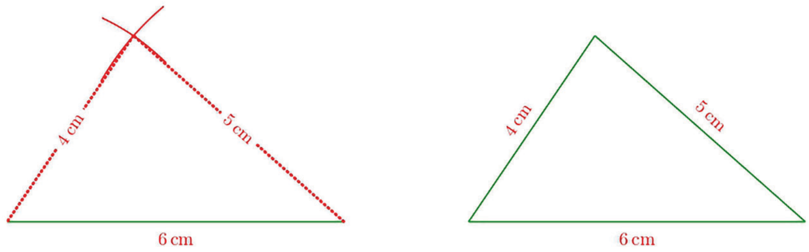
How do we make the left side of such a triangle 6 centimetres?



Can you draw such a triangle with the right side 6 centimetres instead?

Let's look at another problem:

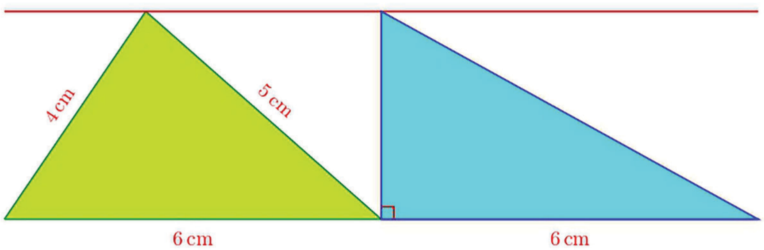
You know how to draw a triangle of sides 4 centimetres, 5 centimetres and 6 centimetres:



To calculate the area of this triangle, we need to measure its height.

Without calculating the area, can you draw a right triangle of the same area?

It is enough that the two triangles must have the same base and height. To make the heights equal, we need only draw the line through the vertex, parallel to the base:



Can you draw a right triangle of the same area with base 4 centimetres?

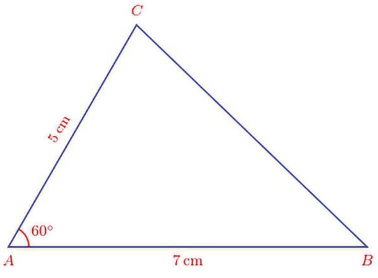
And another with base 5 centimetres?

Now try these problems:

(1) Draw a triangle of sides 3, 4 and 6 centimetres. Draw three different right triangles of the same area.

(2) How many different triangles can be drawn with two sides 8 centimetres, 6 centimetres and area 12 square centimetres? What if the area is to be 24 square centimetres?

(3) Draw the triangle below in the notebook:



Draw triangles ABP, BCQ and CAR of the same area with angles as given below:

(i) $\angle BAP = 90°$ (ii) $\angle CBQ = 60°$

(iii) $\angle ACR = 30°$

11 SQUARES AND RIGHT TRIANGLES

Areas of squares

What is the area of a square with lengths of sides 3metres?

$3 \times 3 = 9$ square metres, isn't it?

What if the lengths of sides is $1\frac{1}{2}$ metres?

$$1\frac{1}{2} \times 1\frac{1}{2} = \frac{3}{2} \times \frac{3}{2} = \frac{9}{4} = 2\frac{1}{4}$$

The area is $2\frac{1}{4}$ square metres.

What about a square of sides 1.2 metres?

$$1.2 \times 1.2 = \frac{12}{10} \times \frac{12}{10} = \frac{144}{100} = 1.44$$

The area is 1.44 square metres.

In all these calculations, we multiplied a number by itself. In other words, we computed the second powers, 3^2, $\left(1\frac{1}{2}\right)^2$, $(1.2)^2$.

The second power of a number is called the *square* of that number. And the operation of finding the square of a number is called *squaring*.

For example,

$$3 \times 3 = 3^2 \qquad \text{Square of 3 (or 3 squared)}$$

$$1\tfrac{1}{2} \times 1\tfrac{1}{2} = \left(1\tfrac{1}{2}\right)^2 \qquad \text{Square of } 1\tfrac{1}{2} \text{ (or } 1\tfrac{1}{2} \text{ squared)}$$

$$1.2 \times 1.2 = (1.2)^2 \qquad \text{Square of 1.2 (or 1.2 squared)}$$

So, we can say that the area of a (geometrical) square is the (arithmetical) square of its side.

Now a question in the other direction:

> To draw a square of area 25 square centimetres, what should be the length of the sides?

The area is the square of the length of the side. So, the question is

The square of which number is 25?

In more detail,

Which number multiplied by itself gives 25?

A little thought gives $5 \times 5 = 25$; so the length of side should be 5 centimetres.

Here we got 5 as the answer to the question, which number squared gives 25. This we can say in another way

5 is the *square root* of 25.

The shorthand notation for the statement, "5 squared is 25" is

$$5^2 = 25$$

The reverse statement is, "5 is the square root of 25". We use the symbol $\sqrt{\ }$ to write this in shorthand; that is,

$$\sqrt{25} = 5$$

We can now write all our earlier computations in reverse:

$9 = 3 \times 3 = 3^2$	Square of 3 is 9	$\sqrt{9} = 3$	Square root of 9 is 3
$2\tfrac{1}{4} = 1\tfrac{1}{2} \times 1\tfrac{1}{2} = \left(1\tfrac{1}{2}\right)^2$	Square of $1\tfrac{1}{2}$ is $2\tfrac{1}{4}$	$\sqrt{2\tfrac{1}{4}} = 1\tfrac{1}{2}$	Square root of $2\tfrac{1}{4}$ is $1\tfrac{1}{2}$
$1.44 = 1.2 \times 1.2 = (1.2)^2$	Square of 1.2 is 1.44	$\sqrt{1.44} = 1.2$	Square root of 1.44 is 1.2

It is not easy to compute the square roots of numbers. For small natural numbers, we can check the squares one by one and find the square root. For slightly larger natural numbers, we can try to factorize them and compute the square root. For example, look at this problem:

What is the length of the side of a square of area 196 square metres?

We can factorize 196 and write

$$196 = 2 \times 2 \times 7 \times 7 = 2^2 \times 7^2$$

We have seen in the lesson, **Repeated Multiplication** that product of powers is the power of the product $(x^n y^n = (xy)^n)$

So,

$$196 = 2^2 \times 7^2 = (2 \times 7)^2 = 14^2$$

Writing this in reverse, we have

$$\sqrt{196} = 14$$

So, the side of the square is 14 metres.

Using this we can also do this problem:

What is the length of the side of a square of area 1.96 square metres?

$$1.96 = \frac{196}{100} = \frac{14^2}{10^2} = \left(\frac{14}{10}\right)^2 = (1.4)^2$$

Writing this in reverse, $\sqrt{1.96} = 1.4$

So, the length of the side is 1.4 metres

Now compute the lengths of the sides of squares with area given below. Write the answers using square roots

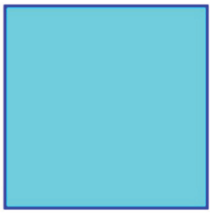
(i) 49 square centimetres (ii) 169 square centimetres

(iii) 400 square centimetres (iv) 225 square centimetres

(v) 1.69 square centimetres (vi) $6\frac{1}{4}$ square metres

Doubling the area

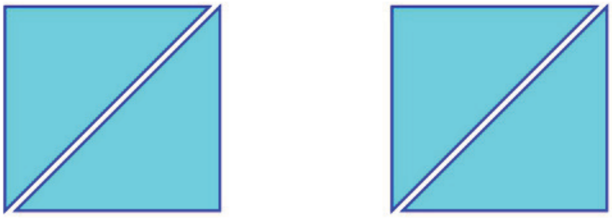
Cut out a square from thick sheet of paper:



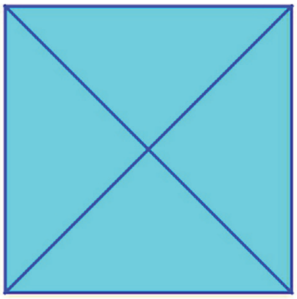
How do you make a square of double the area?

If we measure the length of a side of this square, we can calculate its area as the square of this number; if we double this number and calculate its square root, we get the side of the square of double the area.

There's way to make such a square without any measurement or computation. For that, first cut out another square of the same size. (Remember we have to double the area). Cut both squares along their diagonals:



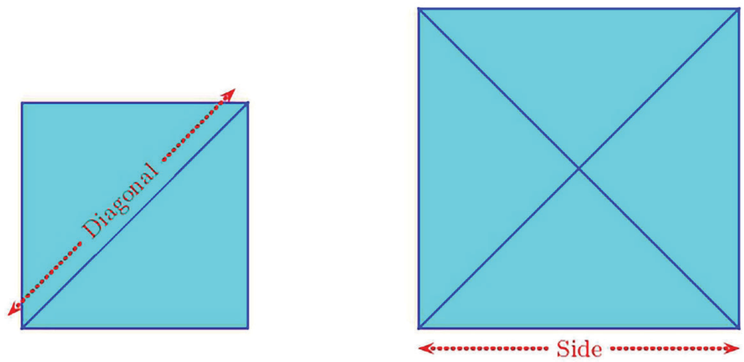
Now arrange the four triangles as below:



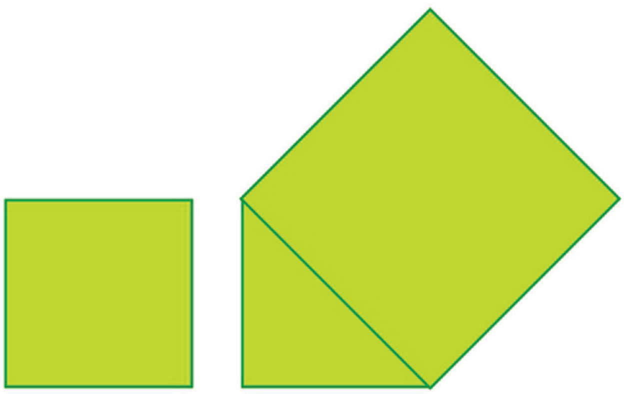
We used all the pieces of the two small squares to make this large square. So, its area is double that of a small square.

There's another way of seeing this. Each of the small squares is made up of two right triangles of the same size; the large square is made up of four such right triangles.

Is the side of the large square equal to the length of some line related to a small square?

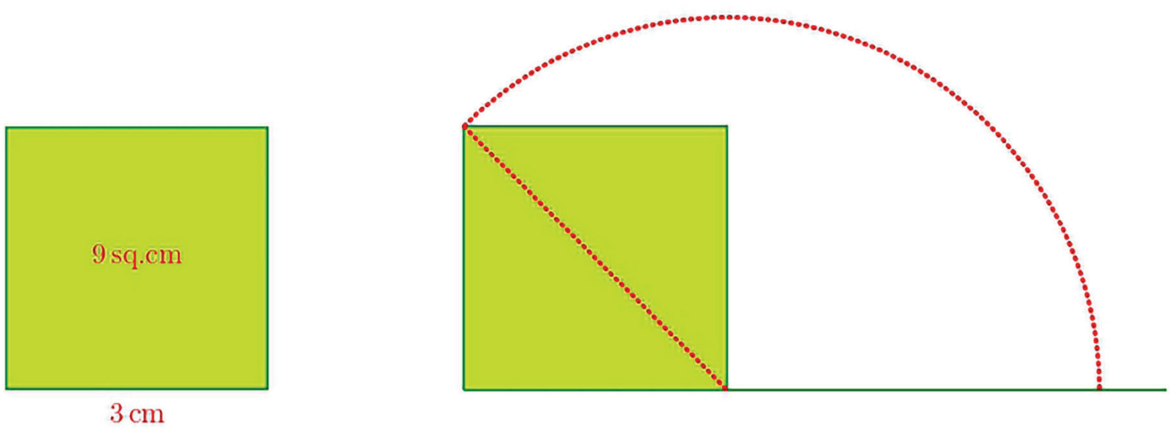


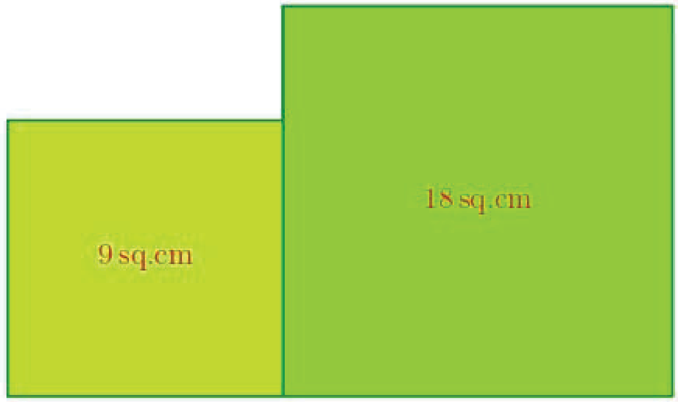
So if we want to just draw a square with double the area of another, we need just draw the square with the diagonal as side:



In GeoGebra, we can draw a square using the  Regular Polygon tool, by clicking on two points and giving the number of sides as 4. Draw a square ABCD like this. Draw the square on a diagonal (Select the Regular Polygon tool and click on D and B) Select the Area tool and click inside each square to get their areas. What is the relation between the areas? Drag B to change the first square and see

If we want to see the squares separately, we just need to one side of the original square and mark the length of the diagonal on it:





Now can't you draw squares of areas as below:

(i) 32 sq.cm

(ii) 50 sq.cm

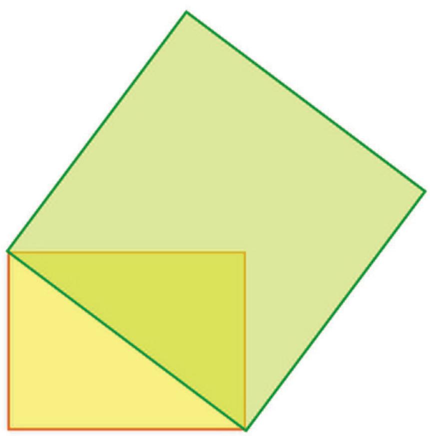
(iii) 12.5 sq.cm

(iv) $24\frac{1}{2}$ sq.cm

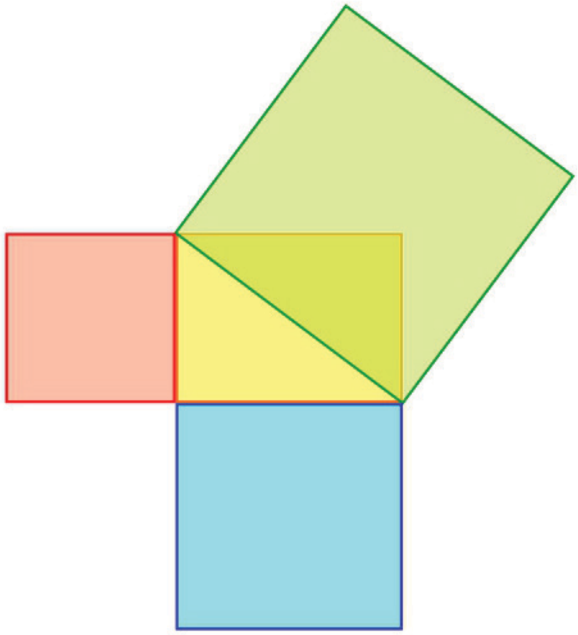
Rectangles and squares

We saw that the area of the square on the diagonal of a square is doubled.

What if we draw the square on the diagonal of a rectangle with unequal sides?

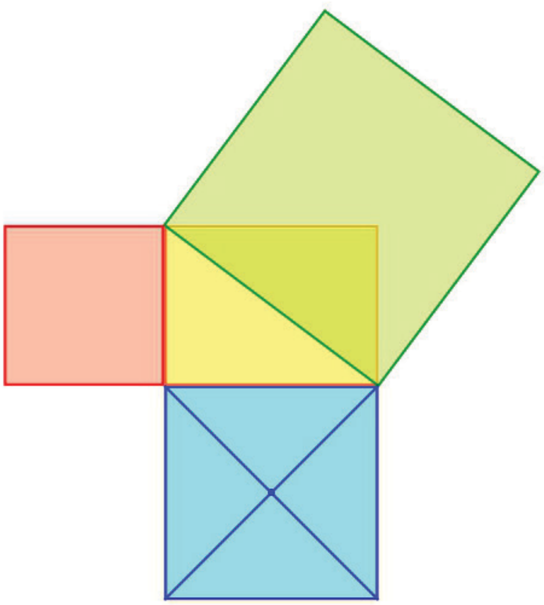


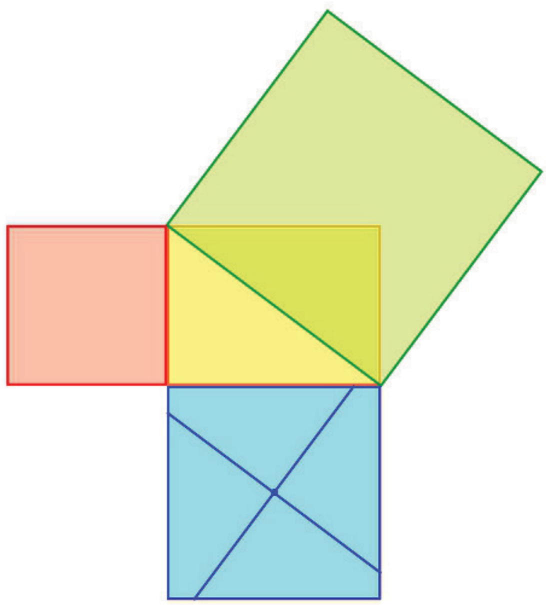
The area of this square also depends on the squares on the sides:



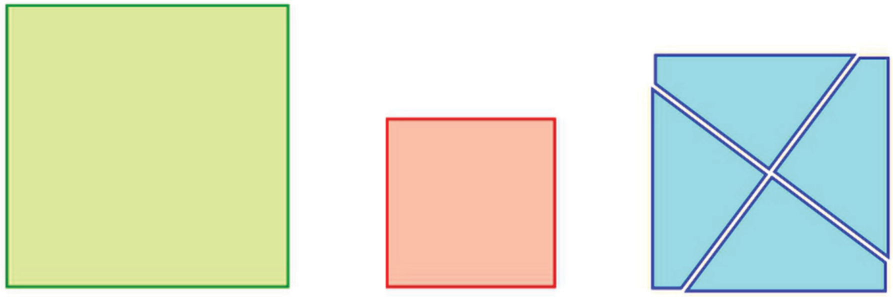
To understand this, draw a rectangle, and squares on its sides and a diagonal as in the picture above, on a thick sheet of paper.

Before cutting it out, we have to draw some more lines. First draw the diagonals of the bottom square and mark the point where they intersect. Then erase the diagonals and draw lines through the point marked, lines parallel to the sides of the largest square:

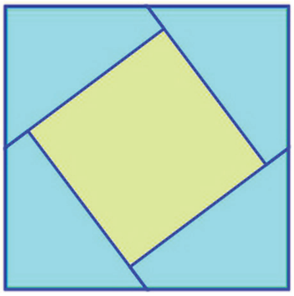


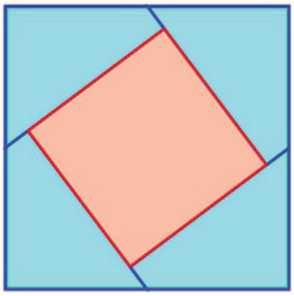


Now cut out the squares; also cut the blue square along the lines drawn inside it:



Arrange the pieces of the blue square and the whole red square within the green square as shown below:





Draw a rectangle in GeoGebra (This can be done easily sing the Grid option.) Use the Regular Polygon tool to draw squares on two adjacent sides and a diagonal. Use the Area tool to get their areas. What is the relation between the areas? Change the lengths of the sides of the rectangle and check

Try this by cutting out different rectangles and squares on their sides and diagonals. Can't the square on the diagonal be exactly filled with squares on the sides?

So, what can we say in general about the areas of these three squares?

The area of the square on the diagonal of a rectangle is equal to the sum of the areas of the squares on the adjacent sides

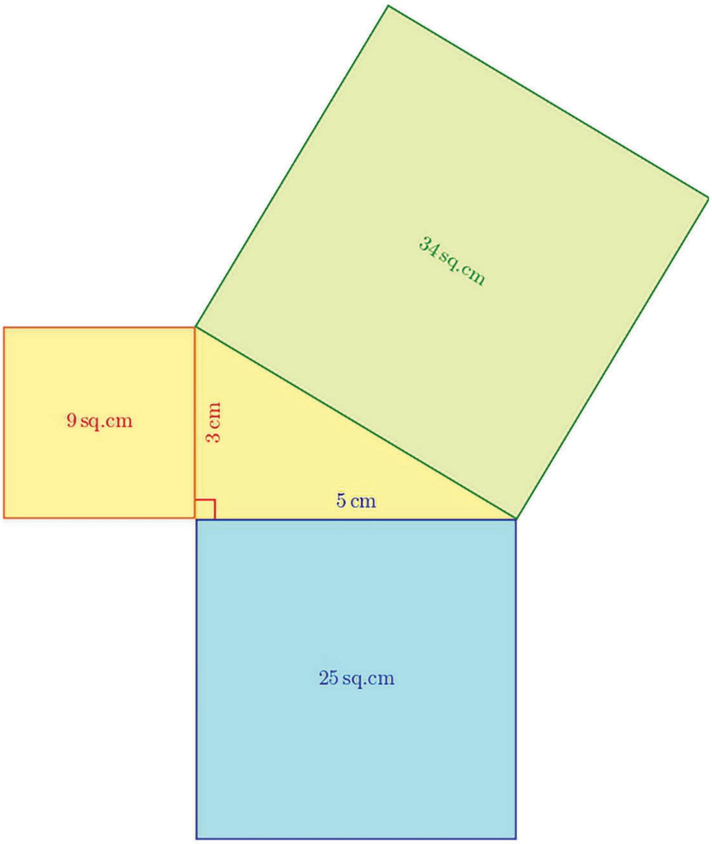
This can be stated in a different way.

In any rectangle, two sides and a diagonal make a right triangle. The adjacent sides of the rectangle are the perpendicular sides of the triangle; and the diagonal is the longest side of the right triangle.

The longest side of a right triangle is called its *hypotenuse*. So the above result can also be stated as below:

The area of the square on the hypotenuse of a right triangle is equal to the sum of the areas of the squares on the perpendicular sides

For example, if the sides of a rectangle are 5 centimetres and 3 centimetres, then the areas of the squares on them are $5^2 = 25$ square centimetres and $3^2 = 9$ square centimetres. So, the area of square on the diagonal is 25 + 9 = 34 square centimetres:



In GeoGebra, create an integer slider n with minimum value 3. For this, select the Slider tool and click on the screen; in the dialog window, select the option Integer. enter 3 as Min and click OK. Now draw a right triangle. select the Regular Polygon tool and click on two vertices. In the dialog window, enter n as Vertices. In this way, we can draw regular n-gons on all three sides of the triangle (equilateral triangle for n = 3, square for n = 4 and so on). Use the Area tool to get their areas. What is the relation between the areas? Change the lengths of the sides of the triangle and the number of sides of the polygons and check

This result is known as the **Pythagoras' Theorem**, named for the mathematician Pythagoras, who lived in ancient Greece.

We can use this to draw squares of specified areas. For example, to draw a square of area 10 square centimetres, we first split 10 as

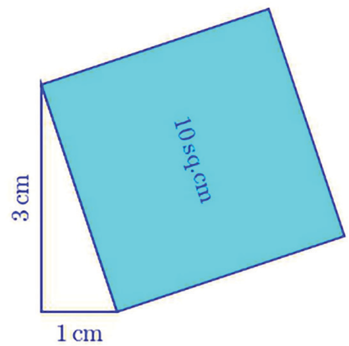
$$10 = 1 + 9 = 1^2 + 3^2$$

This shows that 10 square centimetres is the sum of the areas of a square of side 1 centimetre and a square of side 3 centimetres. So, by Pythagoras' Theorem, if we draw a right triangle with perpendicular sides 1 and 3 centimetres, then the area of the square on the hypotenuse is 10 square centimetres.

Draw squares of areas below in GeoGebra:

(i) 13 (ii) 20 (iii) 26

Check whether the constructions are correct, using the Area tool



What about a square of area 11 square centimetres?

11 cannot be written as the sum of squares of two natural numbers (check!) so we cannot use the above method to draw such a square.

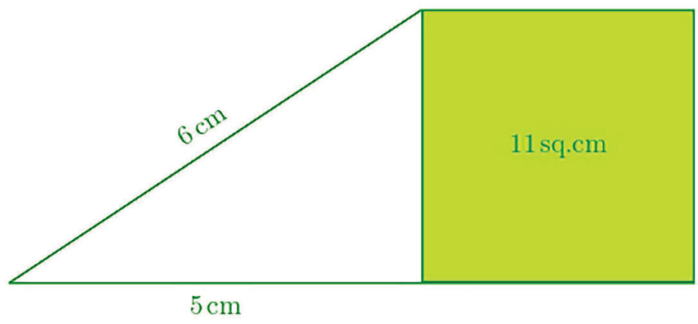
We can think in a different way. According to Pythagoras' Theorem, if from the area of the square on the hypotenuse of a right triangle, we subtract the area of the square on another side, then we get the area of the square on the third side, right?

So, to draw a square of area 11 square centimetres, it is enough to split 11 as the difference of two squares.

A little bit of thinking gives

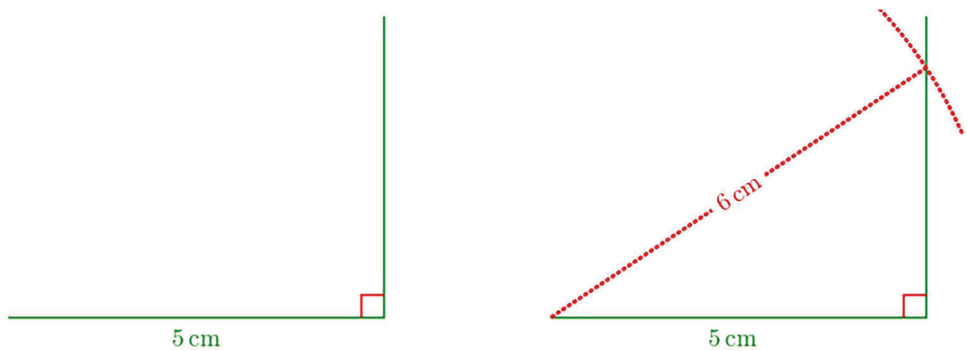
$$11 = 36 - 25 = 6^2 - 5^2$$

So, if we draw a right triangle of hypotenuse 6 centimetres and another side of 5 centimetres, then the area of the square on the third side would be 11 square centimetres:



But how do we actually draw such a triangle?

Draw a line 5 centimetres long and the perpendicular at one end. Next draw a piece of the circle centred at the other end of the line, with radius 6 centimetres. The point where this circle meets the perpendicular is the third vertex of the triangle we want:



Now try these problems:

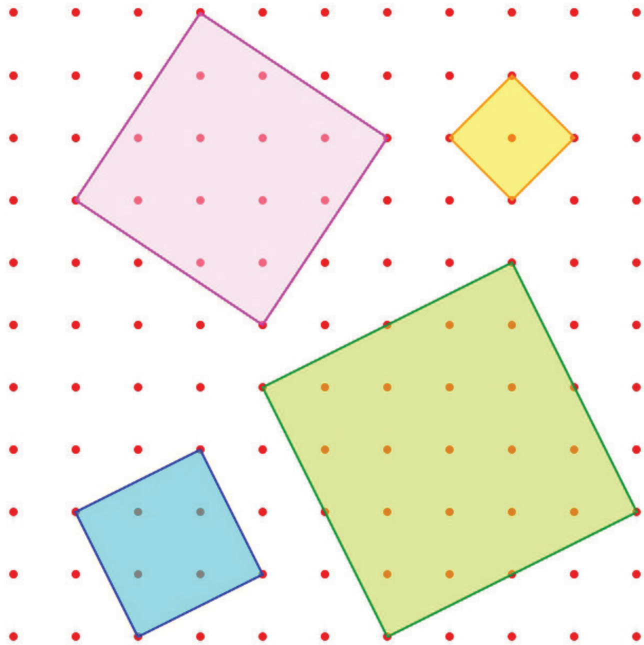
(1) Draw squares of areas given below

(i) 17 square centimetres

(ii) 18 square centimetres

(iii) 19 square centimetres

(2) In the picture below, dots are marked 1 centimetre apart, horizontally and vertically and some of these are joined to make squares:



Calculate their areas.

Line math

Pythagoras' Theorem states a relation between the areas of squares on the sides of a right triangle,

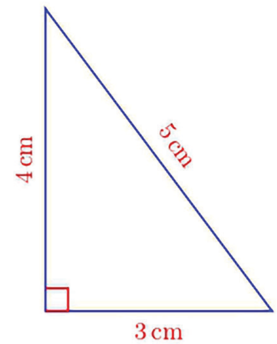
The area of a square is the square of the length of its side; so, Pythagoras' Theorem can also be stated as a relation between the sides of a right triangle:

The square of the hypotenuse of a right triangle is the sum of the squares of its perpendicular sides

For example, if the perpendicular sides of a right triangle are 3 centimetres and 4 centimetres, then the square of its hypotenuse is

$$3^2 + 4^2 = 25$$

So, the hypotenuse of this triangle is 5 centimetres



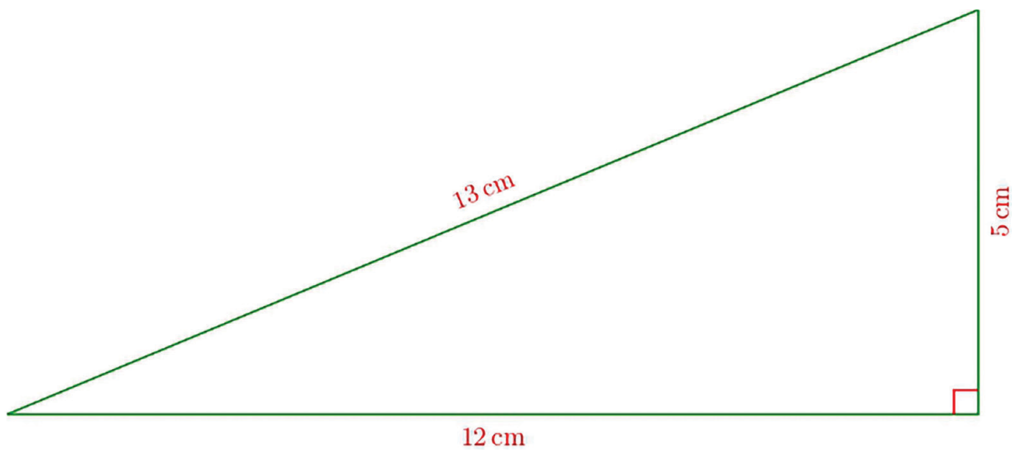
Another question:

The hypotenuse of a right triangle is 14 centimetres and the length of another side is 12 centimetres. What is the length of the third side?

The square of the third side added to the square of its perpendicular side, which is 12, is the square of the hypotenuse, which is 13. So,

Square of the third side = $13^2 - 12^2 = 169 - 144 = 25$.

This gives the length of the third side as 5 centimetres.



Now try these problems

(1) The lengths of two sides of some right triangles are given below. Calculate the length of the third side.

(i) Perpendicular sides 6 centimetres, 8 centimetres

(ii) Perpendicular sides 9 centimetres, 12 centimetres

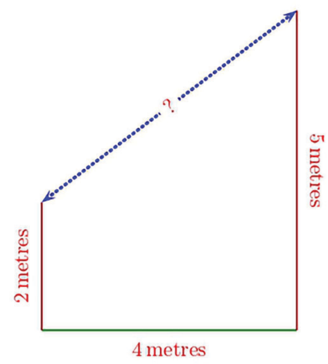
(iii) Perpendicular sides 7 centimetres, 24 centimetres

(iv) Perpendicular sides 14 centimetres, 48 centimetres

(v) Hypotenuse 17 centimetres, another side 15 centimetres

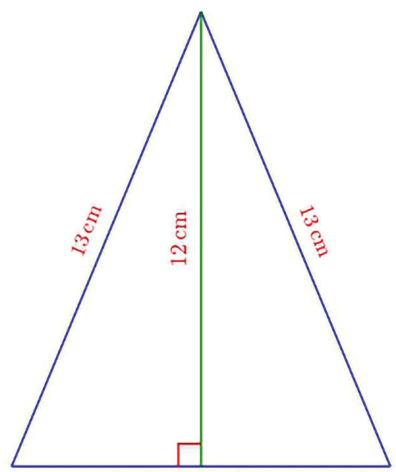
(vi) Hypotenuse 34 centimetres, another side 30 centimetres

(2) Two pillars of heights 2 metres and 5 metres stand 4 metres apart:

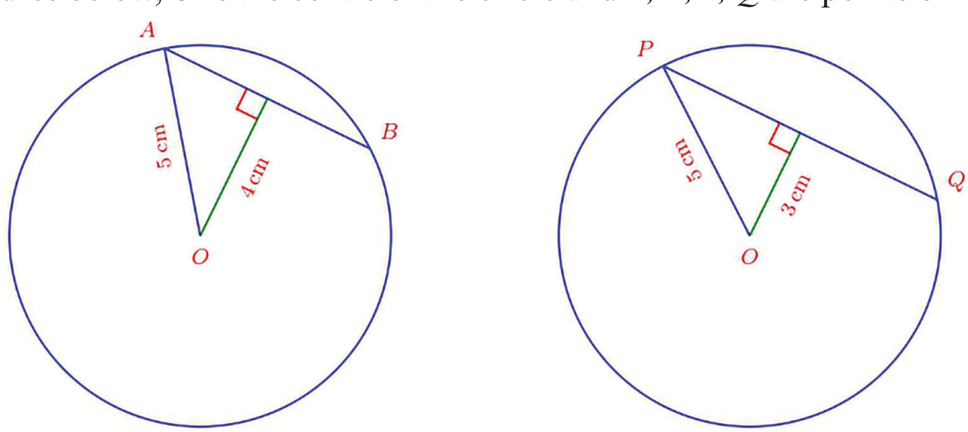


What is the distance between their tops?

(3) Find the length of the bottom side of the triangle drawn below:



(4) In the pictures below, O is the centre of the circle and A, B, P, Q are points on the circle:



Calculate the lengths of the lines AB and PQ.

12 ALGEBRA

Numbers and algebra

We have seen how some facts about measures and numbers can be written in a shorthand form, using algebra.

Let's see some more such instances.

We start with numbers. Adding two consecutive natural numbers give these:

$$1 + 2 = 3$$

$$2 + 3 = 5$$

$$3 + 4 = 7$$

$$4 + 5 = 9$$

Do you see any relation between the first number and the sum?

$$(1 \times 2) + 1 = 3$$

$$(2 \times 2) + 1 = 5$$

$$(3 \times 2) + 1 = 7$$

$$(4 \times 2) + 1 = 9$$

Is this relation true for all numbers?

For example, do we get the same number, if we add 248 and 249 or add 1 to 2 times 248?

We can check by actually adding and multiplying; but that does not explain why this happens.

We want to link 248 + 249 and (2 × 248) + 1

Let's think like this:

- The number 249 in the first operation, is not there in the second operation
- How do we remove the 249 from the first operation?
- How about writing 249 as 248 + 1?

Then we can write like this:

$$249 = 248 + 1$$

$$248 + 249 = 248 + (248 + 1)$$

What next?

The two 248's on the right of this equation together make 2 times 248, right?

$$248 + (248 + 1) = (248 + 248) + 1 = (2 \times 248) + 1$$

This works for any two consecutive natural numbers, doesn't it?

$$432 + 433 = 432 + (432 + 1) = (432 + 432) + 1 = (2 \times 432) + 1$$

So we can write this as a general proposition:

> The sum of two consecutive natural numbers is equal to the sum of twice the smaller number and one.

Next let's see how this can be stated in algebraic language.

Let's write x for the smaller of two consecutive natural numbers. Then what about the next number?

We can write it as y. But here what we want are not two unrelated numbers. What is the relation between one natural number and the next?

Adding one to a natural number gives the next, right?

So, if we use x to denote a natural number, how do we write the next natural number?

1 added to x is $x + 1$

So let's start like this:

(i) x is a natural number

(ii) The next natural number is $x + 1$

(iii) Adding these two gives the number $x + (x + 1)$

Now let's look at the number got by adding one to twice the smaller number:

(i) The smaller number is x

(ii) Twice this is $2x$

(iii) 1 added to this is $2x + 1$

Now can't we write the algebraic form of the proposition above?

$$x + (x + 1) = 2x + 1, \text{ for any natural number } x$$

Did you notice something here? Something we saw in the lesson, **Shorthand Math**:

$$(x + y) + z = x + (y + z) \text{ for any numbers } x, y, z$$

Reading this in reverse,

$$x + (y + z) = (x + y) + z$$

In this, if we take y as x itself and z as 1, what do we get?

$$x + (x + 1) = (x + x) + 1$$

$x + x = 2x$ right? Using this in the above equation, we get

$$x + (x + 1) = 2x + 1$$

And this is the equation of the general result we got above, isn't it?

Looking at it like this, we see another thing: the equation about three numbers x, y and z is true not only for natural numbers, but for fractions also.

So our new result is true for all numbers. The only thing is, instead of *two consecutive numbers*, we should say, *a number and one more than the number.*

Power of algebra

When we add 5 and 6 to get 11, we need not note that 6 is 1 added to 5. But in algebra, when we denote a natural number as x, we can write the next natural number only as $x + 1$.

The advantage of this is that we can write their sum $x + (x + 1)$ as $2x + 1$ and immediately recogize this as one added to twice the smaller number.

Also comparing the equation

$$x + (x + 1) = 2x + 1$$

we got like this and the earlier equation

$$x + (y + z) = (x + y) + z$$

we had got for all numbers, we are led to the thought that the first equation is also true for all numbers.

The sum of a number and one more than it, is equal to sum of twice the number and one.

For example,

$$5\tfrac{1}{2} + \left(5\tfrac{1}{2} + 1\right) = \left(5\tfrac{1}{2} + 5\tfrac{1}{2}\right) + 1$$

$$= \left(2 \times 5\tfrac{1}{2}\right) + 1$$

$$= 11 + 1$$

$$= 12$$

Next take three consecutive natural numbers and add the first and the last:

$$1 + 3 = 4$$
$$2 + 4 = 6$$
$$3 + 5 = 8$$
$$4 + 6 = 10$$

In each of these, do you see any relation between the sum and the middle number we've omitted?

Let's write this relation against each of the above equations:

$1 + 3 = 4$	$2 \times 2 = 4$
$2 + 4 = 6$	$2 \times 3 = 6$
$3 + 5 = 8$	$2 \times 4 = 8$
$4 + 6 = 10$	$2 \times 5 = 10$

To see whether this is true for all such numbers and if so why, let's take another three numbers like these.

Take 27, 28, 29 for example. We want to see if $27 + 29$ and 2×28 give the same result.

As before, let's link (the unwanted) 27 and 29 to (the needed) 28:

$$27 = 28 - 1$$
$$29 = 28 + 1$$

Then we can write

$$27 + 29 = (28 - 1) + (28 + 1)$$

Now let's look at the operations done in $(28 - 1) + (28 + 1)$, one by one:

- Two 28's are added
- 1 is added
- 1 is subtracted

That is,

$$(28 - 1) + (28 + 1) = (2 \times 28) + 1 - 1$$

If we add one to a number and then subtract one, we get the original number back, right?

Thus,

$$(2 \times 28) + 1 - 1 = 2 \times 28$$

Let's write all these equations together:

$$\begin{aligned} 27 + 29 &= (28 - 1) + (28 + 1) \\ &= (2 \times 28) + 1 - 1 \\ &= 2 \times 28 \end{aligned}$$

This works for any three numbers instead of 27, 28, 29, doesn't it?

So, what can we say in general?

> For any three consecutive natural numbers, the sum of the first and the last, is equal to twice the middle number.

Next, how do we state this using algebra?

Here the second operation is stated in terms of the middle number. So, let's denote that by x.

- Middle number is x
- The first number is 1 subtracted from x, that is $x - 1$
- The last number is 1 added to x, that is $x + 1$

So, the above result as an equation is

$$(x - 1) + (x + 1) = 2x$$

As said earlier, the operations in $(x - 1) + (x + 1)$ means adding two x's, then adding 1, and then subtracting 1; in effect, just adding two x's. And this means multiplying x by 2.

$(x - 1) + (x + 1) = 2x$, for any natural number x

We can also state it like this:

$x + z = 2y$ for any consecutive natural numbers x, y, z

But then, we'll have to explain separately, why this is so.

Now try these problems:

(1) Take some sets of three consecutive natural numbers and add all three.

(i) Check if the sum has any relation with any one of the three numbers added.

(ii) Explain why this relation holds for any three consecutive natural numbers.

(iii) Write this relation, first in ordinary language, and then using algebra.

(2) For some sets of four consecutive natural numbers, the sum of the first and the last, and the sum of the middle two, are shown separately below:

1 + 4 = 5	2 + 3 = 5
2 + 5 = 7	3 + 4 = 7
3 + 6 = 9	4 + 5 = 9
4 + 7 = 11	5 + 6 = 11

(i) Explain why such sums are equal for any four consecutive natural numbers

(ii) Write this relation using algebra.

(3) For four consecutive natural numbers, is there any relation between the sum of the first two numbers and the last two numbers ? Explain the reason for this relation and write the relation using algebra. What about the sum of the first and the third numbers and the sum of the second and the fourth ?

Number curiosities

Let's have some more fun with natural numbers.

Write any three consecutive natural numbers in a line:

5 6 7

Add the adjacent pairs and write them above like this:

11 13

5 6 7

Now add these two numbers and write on the very top:

24

11 13

5 6 7

Start with some other sets of three consecutive numbers and build towers like this:

44	80	104
21 23	39 41	51 53
10 11 12	19 20 21	25 26 27

In all such towers, is the top number a multiple of 4?

Which multiple of 4?

We can explain the reason of this also using algebra.

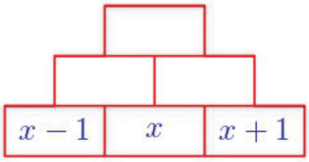
We get at the top, 4 times the middle number of the three we start with. So, let's start by taking it as x.

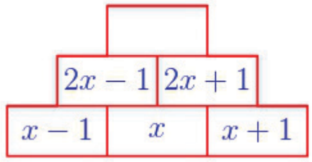
How do we write numbers on the left and right of this?

In three consecutive numbers, the numbers before and after the middle number are one less and one more.

So the bottom line is $x - 1$, x, $x + 1$.

What about the numbers on above that?



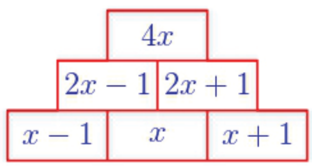


$(x - 1) + x = 2x - 1$

$x + (x + 1) = 2x + 1$

And the number at the top ?

$$(2x - 1) + (2x + 1)$$



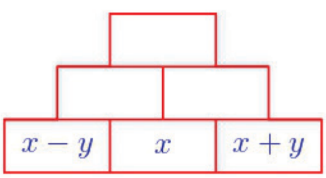
This leads to another thought: what if we take the starting numbers as $x - 2$, x, $x + 2$ instead?

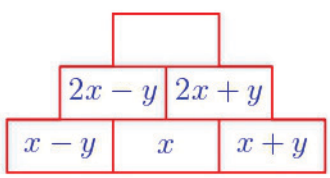
We can mentally calculate the next line: $2x - 2$ and $2x + 2$

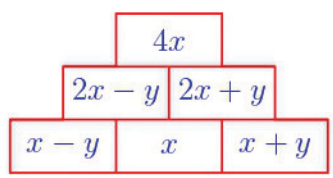
And at the top?

$4x$ again.

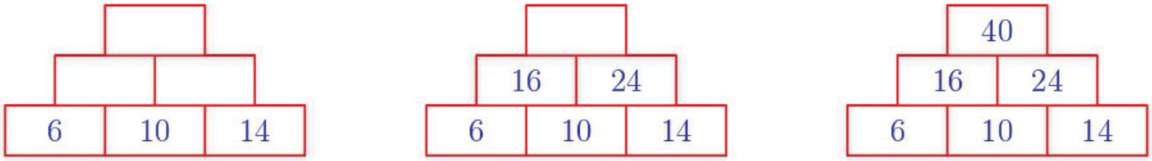
Now suppose we write on the left, any number subtracted from x, and on the right, the same number added to x:







This means we can start with any three numbers equally apart, and build a tower like this; the top number in all these will be four times the middle number:



Now see these four-story towers like the above:

The first one starts with the four consecutive numbers 3, 4, 5, 6

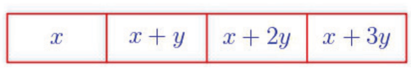
In the second, the four numbers 8, 11, 14 , 17, got by starting with 8 and adding 3 again and again.

Do you see any relation between the top number and the two middle numbers of the first row?

Make some more towers like this, starting with other numbers. In all these, do you get the top number as four times the sum of the middle numbers of the first row?

Let's write this using algebra. We start with x and add y each time to get the other three numbers.

$$x, x + y, x + 2y, x + 3y$$

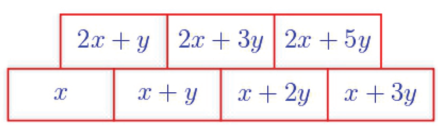


What about the next line?

$$x + (x + y) = 2x + y$$

$$(x + y) + (x + 2y) = 2x + 3y$$

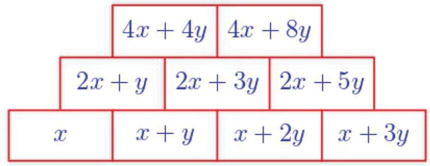
$$(x + 2y) + (x + 3y) = 2x + 5y$$



The third line is

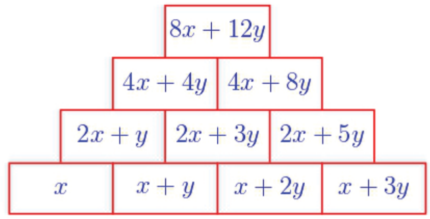
$$(2x + y) + (2x + 3y) = 4x + 4y$$

$$(2x + 3y) + (2x + 5y) = 4x + 8y$$



And the topmost number is

$$(4x + 4y) + (4x + 8y) = 8x + 12y$$



Is the top number equal to four times the sum of the two middle numbers of the first line?

The sum of the middle two numbers of the first line is

$$(x + y) + (x + 2y) = 2x + 3y$$

And four times this is

$$4 \times (2x + 3y) = (4 \times 2x) + (4 \times 3y) = 8x + 12y$$

(Recall that the product of a sum is the sum of the products, as seen in the lesson, **Shorthand Math**)

Had we thought about it a little more, we could have found out the top number just after writing the second line.

The numbers in the second row are

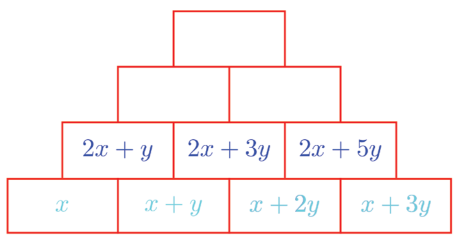
$$2x + y, \quad 2x + 3y, \quad 2x + 5y$$

What if we rewrite them like this:

$2x + y$

$2x + 3y = (2x + y) + 2y$

$2x + 5y = (2x + 3y) + 2y$

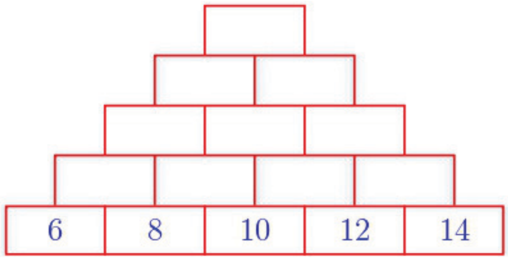


Three numbers, equally at equal distance $2y$ apart.

So the top number of the three-storey tower built on these must be four times the middle number $2x + 3y$, as seen earlier, right?

Now try these problems:

(1) The bottom row of a five-storey number tower (like the three-storey and four-storey towers we have discussed) is shown below:

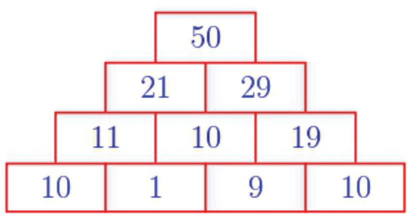


(i) Before actually writing down the other numbers, see if you can guess how many times 10 is the topmost number. Check whether your guess is correct by filling in the other numbers

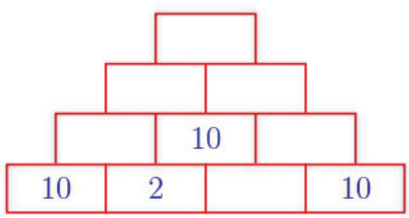
(ii) Explain using algebra that if we start with any five numbers equally apart and make a five-storey tower like this, then the topmost number is a fixed multiple of the middle number.

(iii) Is there any method to determine this without writing all the numbers?

(2) We can make number towers starting with any set of numbers, not necessarily equally apart. For example, look at this tower:



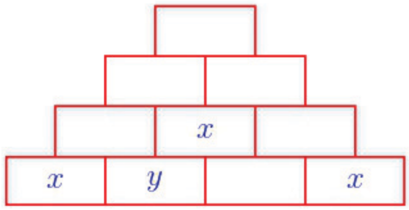
(i) Fill in the empty cells of the tower below:



(ii) In a tower like this, keep the 10's where they are and change the second number of the bottom row to some number other than 1 or 2 and compute the other numbers. Is the topmost number still 50?

(iii) Explain the reason for this using algebra

(iv) Fill in the empty cells of this tower:



(3) Write the numbers from 1 to 100 in rows and columns as in the first picture below. Then draw some 9-cell squares in it, as in the second picture. Mark the middle number of each such square also:

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

1	2	3	4	5	6	7	8	9	10
11	12	13	(14)	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	(63)	64	65	66	67	68	69	70
71	72	73	74	75	76	77	(78)	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Find the following for each of the squares:

(i) The relation between the middle number and the sum of the numbers on its left and right.

(ii) The relation between the middle number and the sum of the numbers on its top and bottom.

(iii) The relation between the middle number and the sums of the pairs of numbers diagonally on its top and bottom.

(iv) The relation between the middle number and sum of all the numbers in the square

Explain all this using algebra

(4) On the calendar of any month, draw 4-cell squares at various positions:

SUN	MON	TUE	WED	THU	FRI	SAT
			1	2	3	4
5	6	7	8	9	10	11
12	13	14	15	16	17	18
19	20	21	22	23	24	25
26	27	28	29	30	31	

(i) Explain using algebra, why the sum of all four numbers in such a square is a multiple of 4.

(ii) Explain using algebra, the relation between this sum and the smallest number in the square.

Multiple and remainder

We have seen in Class 5 that for any two natural numbers, one can be written as the sum of a multiple of the other and a remainder, using division.

For example, dividing 9 by 4, we can write

$$9 = (2 \times 4) + 1$$

On the other hand, dividing 4 by 9, we can write

$$4 = (0 \times 9) + 4$$

Those numbers that can be divided without remainder by 2, we call *even numbers*.

For example,

$$2 = 1 \times 2$$
$$4 = 2 \times 2$$
$$6 = 3 \times 2$$

Thus 2, 4, 6 and so on are even numbers; what about zero?

Since

$$0 = 0 \times 2$$

Zero is also an even number.

In general, we can say this:

Even numbers are those numbers which are divisible by 2

We can also state this using algebra:

Any even number can be written in the form $2n$, where n is one of the numbers 0, 1, 2, 3, . . .

Now what about the numbers which leave a remainder on division by 2?

For example,

$$1 = (0 \times 2) + 1$$
$$3 = (1 \times 2) + 1$$
$$5 = (2 \times 2) + 1$$

These are the *odd numbers.*

In general,

Odd numbers are those that leave a remainder 1 on division by 2

How do we say this using algebra?

Any odd number can be written in the form $2n + 1$, where n is one of the numbers 0, 1, 2, 3, . . .

Writing numbers with special properties in general algebraic form are often useful in recognizing other peculiarities of such numbers.

For example, we can see using examples that the sum of two even numbers is again an even number.

We can use algebra to understand why this is do.

For this, let's take any two even numbers. As seen earlier, they can be written as $2m$ and $2n$, where m and n are either 0 or some natural numbers.

What is their sum?

$$2m + 2n = 2(m + n)$$

In this if we denote the sum $m + n$ by the single letter p, then the sum can be written $2p$. Since m and n are zero or natural numbers, so is their sum p.

So, $2p$ is an even number.

Now what if we add two odd numbers?

Why is the sum an even number?

We can take two odd numbers as $2m + 1$ and $2n + 1$, where m and n are either 0 or some natural numbers.

Their sum is

$$\begin{aligned}(2m + 1) + (2n + 1) &= 2m + 2n + 2 \\ &= 2(m + n + 1)\end{aligned}$$

In this, if we write $m + n + 1$ as p, then our sum is $2p$ and p is a natural number.

So the sum is a even number.

In the same way, can you explain using algebra, why the sum of an even number and an odd number is an odd number?

We split the natural numbers into two sets, even and odd, based on the remainders got on division by 2.

What if we do this based on the remainders got on division by 3?

Set of numbers	Speciality	Algebraic form
0, 3, 6, 9, ...	Remainder 0 on division by 3	$3n \quad (n = 0, 1, 2, 3...)$
1, 4, 7, 10, ...	Remainder 1 on division by 3	$3n + 1 \quad (n = 0, 1, 2, 3...)$
2, 5, 8, 11, ...	Remainder 2 on division by 3	$3n + 2 \quad (n = 0, 1, 2, 3...)$

Now try these problems:

(1) Add a number leaving remainder 1 on division by 3, and a number leaving remainder 2 on division by 3. Explain using algebra, why the sum of any two such numbers is divisible by 3

(2) The numbers 12, 23, 34, . . . are got by starting with 12 and adding 11 again and again.

(i) What is the remainder if any such number is divided by 11?

(ii) Write the general algebraic form of all these numbers

(iii) Is 100 among these numbers? What about 1000?

(3) The numbers 21, 32, 43, . . . are got by starting with 21 and adding 11 again and again

(i) What is the remainder if any such number is divided by 11?

(ii) Write the general algebraic form of all these numbers.

(iii) Is 100 among these numbers? What about 1000?

Digits and numbers

We can write all two-digit numbers in rows and columns like this:

10	11	12	13	14	15	16	17	18	19
20	21	22	23	24	25	26	27	28	29
30	31	32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47	48	49
50	51	52	53	54	55	56	57	58	59
60	61	62	63	64	65	66	67	68	69
70	71	72	73	74	75	76	77	78	79
80	81	82	83	84	85	86	87	88	89
90	91	92	93	94	95	96	97	98	99

The numbers in the first row are got by adding to 10, the numbers from 0 to 9. So, their general algebraic form can be written

$$10 + n \quad (n = 0, 1, 2, ..., 9)$$

What about numbers in the second row?

We can write them as

$$20 + n \quad (n = 0, 1, 2, ..., 9)$$

or as

$$(2 \times 10) + n \quad (n = 0, 1, 2, ..., 9)$$

We can write the algebraic form of numbers in each row like this:

n	0	1	2	3	4	5	6	7	8	9
$(1 \times 10) + n$	10	11	12	13	14	15	16	17	18	19
$(2 \times 10) + n$	20	21	22	23	24	25	26	27	28	29
$(3 \times 10) + n$	30	31	32	33	34	35	36	37	38	39
$(4 \times 10) + n$	40	41	42	43	44	45	46	47	48	49
$(5 \times 10) + n$	50	51	52	53	54	55	56	57	58	59
$(6 \times 10) + n$	60	61	62	63	64	65	66	67	68	69
$(7 \times 10) + n$	70	71	72	73	74	75	76	77	78	79
$(8 \times 10) + n$	80	81	82	83	84	85	86	87	88	89
$(9 \times 10) + n$	90	91	92	93	94	95	96	97	98	99

We can also see that in these, n is the digit in the one's place in each of these numbers.

Now if we scan the columns instead of the rows, we can see that the digit in the ten's place in each column changes from 1 to 9. So, if we write the digit in the ten's place as m, then all these numbers can be written in the algebraic form $10m + n$. Thus the general algebraic form of all two-digit numbers is

$$10m + n \qquad (m = 1, 2, ..., 9; \quad n = 0, 1, 2, ..., 9)$$

For example, taking $m = 2$ and $n = 5$, we get the number $(2 \times 10) + 5 = 25$; on the other hand, to get 49, we take $m = 4$ and $n = 9$.

Now let's look at a problem using this. Take any two-digit number and the two-digit number got by reversing its digits; subtract the smaller from the larger. For example,

$$21 - 12 = 9$$

$$64 - 46 = 18$$

$$52 - 25 = 27$$

Meaning of letters

In the algebraic form $10m + n$ of all two digit numbers, the letters m and n can be seen in two ways:

m is

- the digit in the ten's place of the number
- the quotient on dividing the number by 10

n is

- the digit in the one's place of the number
- the remainder on dividing the number by 10

Do you see any speciality of the numbers got as differences?

Let's check this using algebra

Let's write the larger of the two numbers got like this as $10m + n$.

To get the number on reversing the digits, we need only interchange m and n.

That is, the reversed number is $10n + m$

Now to subtract the smaller from the larger, we must subtract $10n + m$ from $10m + n$. For this, we can first subtract $10n$ and then m (Recall the equation $x - (y + z) = (x - y) - z$, seen in the lesson **Shorthand Math**)

That is,

$$(10m + n) - (10n + m) = (10m + n - 10n) - m$$

In this, what does $10m + n - 10n$ mean?

To the number $10m$, first add the number n and then subtract 10 times n; so in effect we subtract only 9 times n. That is,

$$10m + n - 10n = 10m - 9n$$

(We can do this in another way also. We have seen that if $y > z$, then $(x - y) + z = x - (y - z)$ in the lesson **Shorthand Math**. Writing $10m + n - 10n$ as $10m - 10n + n$, we can use this to write.

$$\begin{aligned} 10m - 10n + n &= 10m - (10n - n) \\ &= 10m - 9n \end{aligned}$$

getting the same equality as before)

Now we can find the difference of the two-digit number and its reverse:

$$\begin{aligned} (10m + n) - (10n + m) &= (10m + n - 10n) - m \\ &= (10m - 10n + n) - m \\ &= (10m - 9n) - m \\ &= 10m - m - 9n \\ &= 9m - 9n \\ &= 9(m - n) \end{aligned}$$

Thus we get the difference as a multiple of 9

We also see another thing here. $m - n$ is the difference of the digits of the number, right? So, we see not only that the difference between a two-digit number and the number with its digits reversed, is a multiple of 9, but also that it is the difference of the digits multiplied by 9

Now try these problems:

(1) Add any two-digit number and the number got by reversing the digits. Explain using algebra, why all such sums are multiples of 11.

(2) From a two-digit number, subtract the sum of the digits. Explain using algebra why all such differences are multiples of 9.

(3) (i) Write the algebraic form of all three-digit numbers.

(ii) Take any three-digit number and the number got by reversing its digits. Subtract the smaller from the larger. Explain using algebra, why all such differences are multiples of 99.

(iii) From a three-digit number, subtract the sum of the digits. Explain using algebra why all such differences are multiples of 9.

13 Percents

Percents and fractions

We have seen percents and some instances of its use in class 6. For example, look at this problem:

During *Onam*, there is 30% reduction in the price of handloom cloths. What is the reduction for 3750 rupees worth of cloths?

The reduction in price is of 30 rupees for every 100 rupees. So we can calculate like this:

Reduction for 3000 rupees	:	900 rupees
Reduction for 700 rupees	:	210 rupees
Reduction for 50 rupees	:	15 rupees
Total reduction	:	1125 rupees

Instead of splitting the computation as above, we can do it at one stroke. For that we reason like this:

- For any price, the reduction is 30 times the number of 100's in it.
- That is, 30 times $\frac{1}{100}$ of the price, which means $\frac{30}{100}$ of the price.
- $\frac{30}{100} = \frac{3}{10}$
- Thus reduction is $\frac{3}{10}$ of the price.
- $\frac{3}{10}$ of 3750 is

$$\frac{3}{10} \times 3750 = \frac{3 \times 3750}{10} = \frac{3 \times 375 \times 10}{10} = 375 \times 3 = 1125$$

- The reduction for 3750 rupees is 1125 rupees

We can also use decimal form of $\frac{3}{10}$.

$$0.3 \times 3750 = 1125.0 = 1125$$

(This is the easier way to do computations using a calculator)

Do you remember another instance where percents are used?

If we deposit money in banks and withdraw it after a specified time, we get some extra amount called the interest (The extra amount when loans are repaid is also interest).

Look at this problem:

> In a co-operative bank, fixed deposits for a year are given 9% interest. If 7500 rupees are deposited, how much would be got after one year?

The interest is 9 rupees for every 100 rupees. What we have to calculate is how much interest would be got for 7500 rupees.

Can't we do this also using fractions, as in the first example?

9 is $\frac{9}{100}$ of 100

So what can we say, instead of saying interest is 9 rupees for every 100 rupees?

The interest is $\frac{9}{100}$ of the amount deposited.

$\frac{9}{100}$ of 7500 is

$$\frac{9}{100} \times 7500 = \frac{9 \times 75 \times 100}{100} = 9 \times 75 = 675$$

Using decimals, the computation is

$$0.09 \times 7500 = 675.00 = 675$$

So, if 7500 rupees is deposited, then the amount got after one year, including interest, is 7500 + 675 = 8175 rupees

We did these problems by converting the percents into fractions. In the tables below, we give the fractional forms of some commonly used percents. Can you fill in the second table?

Percent	As fraction	Reduced form
50%	$\frac{50}{100}$	$\frac{1}{2}$
25%	$\frac{25}{100}$	$\frac{1}{4}$

Percent	As fraction	Reduced form
75%		
20%		

Now try these problems:

(1) An electronics manufacturer sells some of their older models at reduced prices, as shown in the table below:

Device	Original price	Reduction
Laptop	65000	10%
Mobile phone	25000	20%
Smartwatch	12000	30%

Compute the current price of each.

(2) A businessman donates 2% of his monthly profits to charity. In a certain month he got 25000 rupees as profit. How much of this did he donate to charity?

(3) (i) Persons earning between two and a half lakhs and five lakhs annually, must pay five percent of this as income tax. How much income tax should a person with annual income three and a half lakhs pay?

(ii) Persons earning between five and ten lakhs should pay five percent of five lakhs and twenty percent of the excess over five lakhs. How much income tax should a person with annual income seven and a half lakhs pay?

Some other percents

Percents are used not only in money matters, but in some other instances also. For example, what does it mean to say

450 kids took an exam and 80% passed it

This means $\frac{80}{100}$ of those who took the exam passed it. Since

$$\frac{80}{100} = \frac{8}{10} = \frac{4}{5}$$

we can also say $\frac{4}{5}$ of those who took the exam passed it.

So what is the actual number of kids who passed the exam?

$$\frac{4}{5} \times 450 = 4 \times 90 = 360$$

We can also have a question in reverse:

360 persons passed a test. This is 75% of those who took the test. How many took the test?

$\frac{75}{100} = \frac{3}{4}$ of those who took the test passed it. So, the number of persons who took the test is $\frac{4}{3}$ times the number of persons who passed the test. (The lesson, **Reciprocals**.) So, the number of persons who took the test is

$$\frac{4}{3} \times 360 = 4 \times 120 = 480$$

Now try these problems:

(1) Of the 50 teachers in a school, 80% are women. How many female teachers are there in the school?

(2) 1450 persons voted in an election contested by two persons. The winner got 52% of the votes

(i) How many votes did he get?

(ii) By how many votes did he win?

(3) 1200 kids took an exam and 65% of them got A grade. How many are they?

(4) There are 32 coconut palms in a compound. This is 50% of the total number of trees in the compound. What is the total number of trees?

(5) A person spends 8400 rupees a month for food. It is 25% of his monthly earnings. What is his monthly earnings?

Less and more

Let's slightly change the problem posed at the beginning of the lesson:

During *Onam*, there is 30% reduction in the price of handloom cloths. How much should be paid for cloths worth 3250 rupees?

How do we do this?

As before, we can calculate the reduction first:

$$\frac{3}{10} \times 3250 = 3 \times 325 = 975$$

Next we can calculate the amount to be paid as 3250 − 975 = 2275 rupees

Instead of first calculating the reduction and then subtracting it, we can directly compute the amount to be paid.

We reason like this:

- $\frac{3}{10}$ of the price is to be subtracted
- Subtracting $\frac{3}{10}$ of a number from it leaves $\frac{7}{10}$ of the number
- So, amount to be paid is $\frac{7}{10}$ of the price
- $\frac{7}{10}$ of 3250 is

$$\frac{7}{10} \times 3250 = 7 \times 325 = 2275$$

This can be stated like this also:

- 30% reduction means 30 rupees less for every 100 rupees
- So, for 100 rupees worth of cloths, only 70 rupees need be paid
- This means 70% of the price

Let's look at another problem done earlier.

A laptop priced originally at 65000 rupees is now sold for 10% less. What is the current price?

As we have seen just now, 10% less means 90% of the original

So the current price of the laptop is 90% of the original

$$\frac{90}{100} \times 65000 = 90 \times 650 = 9 \times 65 \times 100 = 58500$$

Thus the current price is 58500 rupees

Look at this problem:

A scooter manufacturer has decided to increase the price by 3% from next month. For a scooter now selling for 80000 rupees, what would be the price next month?

We can first compute the increase in price next month, and then add it to the current price to get the price next month.

Or we can reason like this:

- The increase is $\frac{3}{100}$ of the current price
- $\frac{3}{100}$ of a number added to it, makes it $\frac{103}{100}$ times the original
- So the new price is $\frac{103}{100}$ times the current price
- $\frac{103}{100}$ times 80000 is

$$\frac{103}{100} \times 80000 = 103 \times 800 = 103 \times 8 \times 100 = 82400$$

Thus the price in the next month would be 82400 rupees.

Instead of $\frac{103}{100}$ times a number, we can also say 103% of the number.

Another problem:

The width of a rectangle is increased by 10% and the height decreased by 10%, to make a new rectangle. What is the change in area?

Increasing by 10% means becoming 110%; that is, $\frac{110}{100} = \frac{11}{10}$ of the original

$$\text{New width} = \frac{11}{10} \times \text{Old width}$$

Decreasing by 10% means becoming 90%; that is, $\frac{90}{100} = \frac{9}{10}$ of the original

$$\text{New height} = \frac{9}{10} \times \text{Old height}$$

Now to compute the new area, we need to calculate the product of the new width and new height:

$$\begin{aligned}\text{New area} &= \text{New width} \times \text{New height}\\ &= \left(\tfrac{11}{10} \times \text{Old width}\right) \times \left(\tfrac{9}{10} \times \text{Old height}\right)\\ &= \frac{11}{10} \times \frac{9}{10} \times (\text{Old width} \times \text{Old height})\\ &= \frac{99}{100} \times \text{Old area}\\ &= 99\% \text{ of the old area}\end{aligned}$$

So, the area is reduced by 1%

Now try these problems:

(1) A bicycle originally priced at 4000 rupees is now sold for 15% less. What is the current price?

(2) The monthly salary of a person was 30000 rupees last year. This year, he got an 8% raise. What is his monthly salary now?

(3) If the height and width of a rectangle are reduced by 10% each, by what percent would the area be reduced?

(4) If the height and width of a rectangle are increased by 10% each, by what percent would the area be increased?

Fractions and percents

See this headline in School News:

SSLC results declared
Pass percent 99.70%

What does 99.70% mean?

Percent means hundredths multiplied by a specific number. For example 99% of a number means $\frac{99}{100}$ of that number, and that is 99 times $\frac{1}{100}$ of that number.

So, 99.70% means 99.70 times $\frac{1}{100}$. That is,

$$99.70 \times \frac{1}{100} = \frac{9970}{100} \times \frac{1}{100} = \frac{997}{1000}$$

This means, instead of saying $\frac{997}{1000}$ of a number, we can say 99.7% (or 99.70%) of that number.

Thus what the news above says is that the number of children who passed the SSLC exam is $\frac{997}{1000}$ of the number of children who took the exam.

Instead of the decimal form of a part, fractional forms are also sometimes used as percents.

For example, employees in public sector undertakings are given an extra amount, in addition to salary, every year. It is called bonus. Usually it is $8\frac{1}{3}$%. What fraction of the annual salary is this?

As in the last problem, this means $8\frac{1}{3}$ times $\frac{1}{100}$. That is,

$$8\frac{1}{3} \times \frac{1}{100} = \frac{25}{3} \times \frac{1}{100} = \frac{25}{300} = \frac{1}{12}$$

So $8\frac{1}{3}$% of a number is $\frac{1}{12}$ of that number.

Can you write as fractions, the parts given as percents below?

Part	
As percent	As fraction
$12\frac{1}{2}$%	
$6\frac{1}{4}$%	
$33\frac{1}{3}$%	
$66\frac{2}{3}$%	
$16\frac{2}{3}$%	
$83\frac{1}{3}$%	

How do we convert into a fraction, a part given as percent?

For example, $83\frac{1}{3}\%$ of a number is converted to a fraction of that number by computing $83\frac{1}{3}$ times $\frac{1}{100}$.

$$83\frac{1}{3} \times \frac{1}{100} = \frac{250}{3} \times \frac{1}{100} = \frac{250}{300} = \frac{5}{6}$$

This we can also write as

$$\frac{5}{6} = \frac{1}{100} \times 83\frac{1}{3}$$

This means

$$\frac{5}{6} \text{ is } \frac{1}{100} \text{ of } 83\frac{1}{3}$$

In general we can state this as follows:

To convert a part expressed as a percent to the fractional form, $\frac{1}{100}$ of that number is to be computed

For example, the fractional form of the part given as $12\frac{1}{2}\%$ can be computed like this:

$$\frac{1}{100} \times 12\frac{1}{2} = \frac{1}{100} \times \frac{25}{2} = \frac{25}{200} = \frac{1}{8}$$

Thus $12\frac{1}{2}\%$ of a number is $\frac{1}{8}$ of that number.

Then there is the reverse problem: how do we write $\frac{1}{3}$ of a number as a percent of that number?

We think like this:

- $\frac{1}{3}$ (of a number) is $\frac{1}{100}$ of the percent (of that number)
- So, the percent is 100 times $\frac{1}{3}$
- That is,

$$100 \times \frac{1}{3} = \frac{100}{3} = 33\frac{1}{3}$$

Thus $\frac{1}{3}$ of a number is $33\frac{1}{3}\%$ of that number.

We can convert any part expressed as a fraction to a percent like this

To convert a part expressed as a fraction to a percent, 100 times the fraction must be computed

Now look at this problem:

> 120 children took a test and 110 of them passed it. What percent of those who took the test passed it?

The number of children who passed the test is $\frac{110}{120} = \frac{11}{12}$ of those who took the test.

To express this as a percent, we have to calculate 100 times $\frac{11}{12}$.

$$100 \times \frac{11}{12} = \frac{100 \times 11}{12} = \frac{25 \times 11}{3} = \frac{275}{3} = 91\frac{2}{3}$$

Thus $91\frac{2}{3}\%$ of those who took the test passed it.

In trade, the profit or loss is often stated as a percent of the amount spent in buying things.

Look at this problem:

If a table bought for 10000 rupees is sold for 10550 rupees, what is the profit percent?

The actual profit is 10550 − 10000 = 550 rupees. This is $\frac{550}{10000} = \frac{55}{1000}$ of the price for which it was bought.

To express this as a percent, we find 100 times this fraction, as in the earlier examples:

$$100 \times \frac{55}{1000} = \frac{55}{10} = 5\frac{1}{2}$$

So the profit is $5\frac{1}{2}\%$.

We can also write like this:

$$\frac{55}{10} = 5.5$$

so that profit is 5.5%.

Now can't you do these problems?

(1) In a school there are 750 students and 450 of them are girls. What is the percent of girl students?

(2) A person who earns 30000 rupees a month spends 8000 rupees on food. What percent of the earnings is this?

(3) If a bicycle bought for 4500 rupees had to be sold for 4000 rupees, what is the loss percent?

(4) 1600 persons voted in an election and the winner got 900 votes. What percent of the total votes is this?

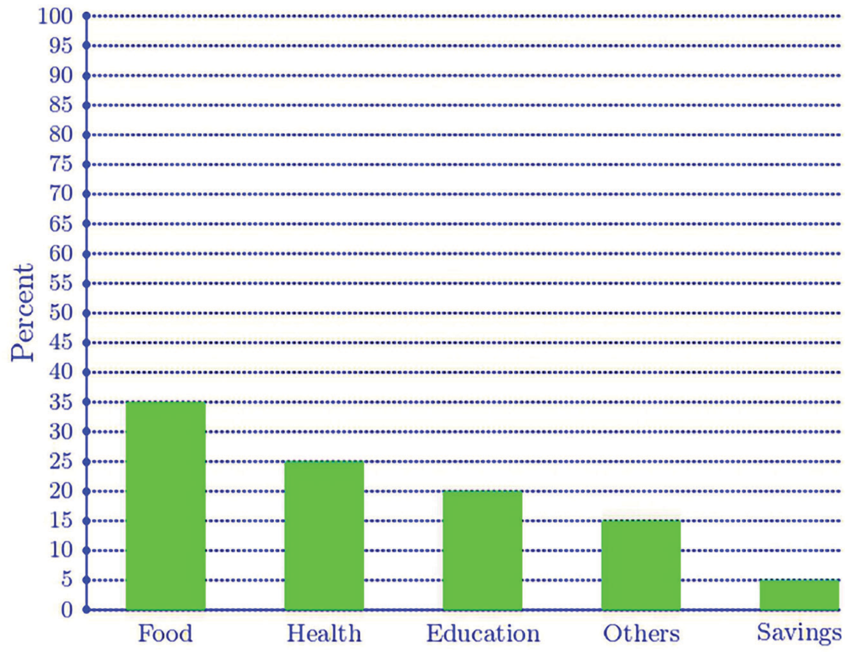
(5) Ajayan's salary is 25% more than Sajayan's salary. By what percent of Ajayan's salary is Sajayan's salary less?

14 Data Pictures

The table below shows the various expenses in a household, as percents of the total:

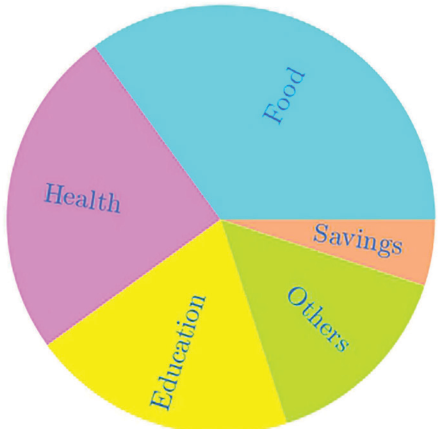
Item	Amount
Food	35%
Health	25%
Education	20%
Others	15%
Savings	5%

We have seen in class 5, how this can be shown as a bar graph:



From this picture, we can easily see the differences in the expenses for different things. But we cannot see at a glance, approximately what fraction of the whole are the expenses for various items or what fraction of the expense for one item is another.

Now look at another representation of the same data:



From this picture, we cannot only see that the most amount is spent on food, but also that it is about a third of the total expenses. Again, we can see that a quarter of the total expenses is related to health.

Such a picture is called a *pie chart*

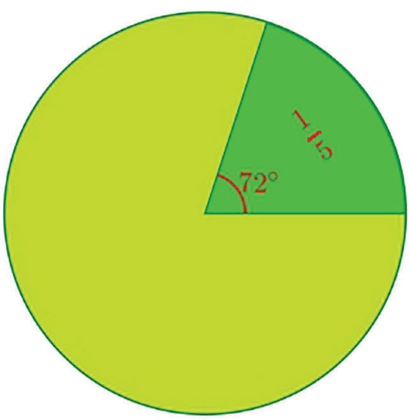
How is this picture drawn?

Food for thought

Pie is a baked dish much popular in western countries:

It is shared by cutting into pieces as in the picture. Hence the name pie chart

Remember how we marked $\frac{1}{5}$ of a circle, by drawing an angle of $\frac{1}{5} \times 360° = 72°$ at the centre?

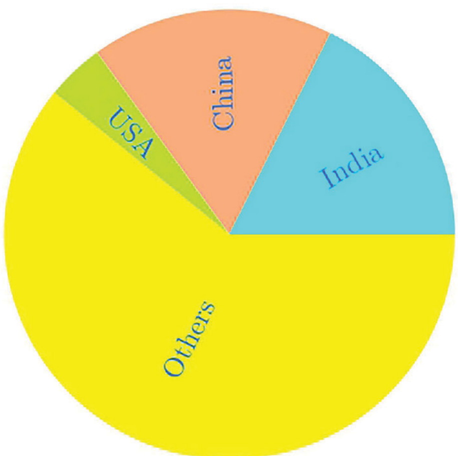


In our table of expenses, the expenditure on food is $\frac{35}{100} = \frac{7}{20}$ of the total, right? To mark this part of a circle, what angle should we take at the centre?

$$\frac{7}{20} \times 360° = 126°$$

Can you compute the angle for each of the expenses and draw the pie chart?

Let's look at another pie chart. The most populous countries of the world are first India, then China and the third the United States of America. The pie chart below shows these and the population of the remaining countries as parts of the total population of the world:

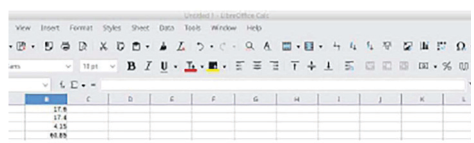


Can you answer these questions using this picture?

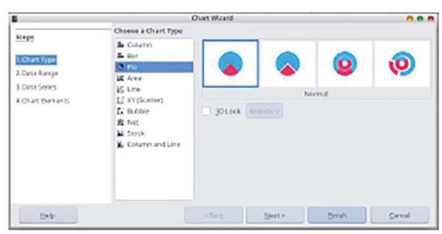
- Approximately what fraction of the world population is that of India?
- What about the population of China?
- Approximately what fraction of the population of India is that of the USA?
- Approximately what fraction of the world population is the combined population of India, China and the USA?

Computer pictures

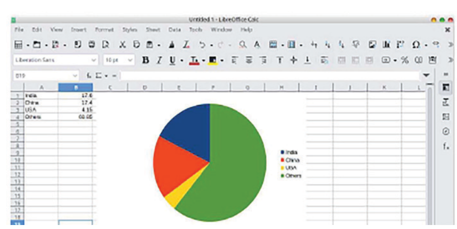
The populations of India, China and the USA are computed as 17.6%, 17.4% and 4.15%. It is not easy for us to draw a pie chart using such numbers. But we can use a computer to do it. First we type this data in Libre Office Calc:



Then we click on the icon on the top right and in the dialogue window which opens up, choose the option Pie Chart in the Chart Type menu:



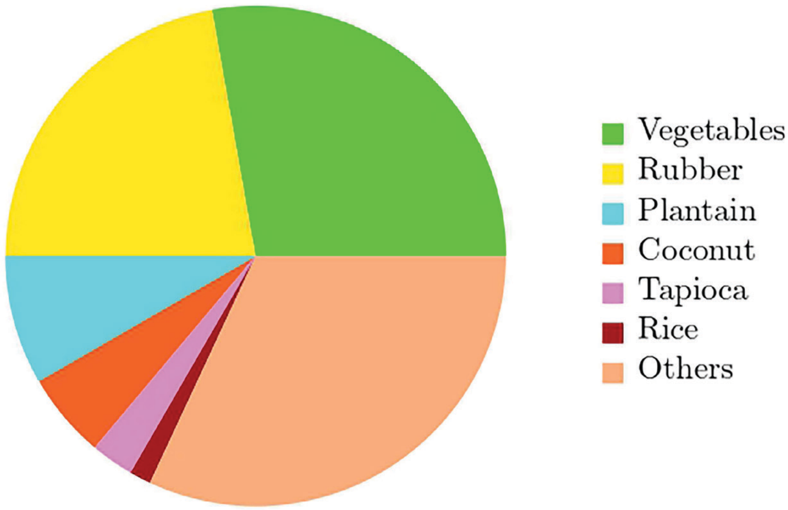
We get the pie chart below:



What other facts can you get from this pie chart?

Now try these problems:

(1) The pie chart below shows how the total cultivable land in a panchayat is utilised for different kinds of crops:



(i) For which crop is most of the land used? Approximately what fraction of the total is this?

(ii) For which crop is the least amount of land used? Approximately what fraction of the total is this?

(iii) Approximately what fraction of the total is used for rubber?

(2) In a class of 40 students, 20 use the school bus, 15 walk to the school and 5 ride bicycles to school. Draw a pie chart showing this.

(3) In a class 25% got A grade in a math test, 40% got B grade, 20% got C grade and the remaining D grade. Draw a pie chart showing this.

Notes

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Notes

Notes

CONSTITUTION OF INDIA

Part IV A

FUNDAMENTAL DUTIES OF CITIZENS

ARTICLE 51 A

Fundamental Duties- It shall be the duty of every citizen of India:

(a) to abide by the Constitution and respect its ideals and institutions, the National Flag and the National Anthem;

(b) to cherish and follow the noble ideals which inspired our national struggle for freedom;

(c) to uphold and protect the sovereignty, unity and integrity of India;

(d) to defend the country and render national service when called upon to do so;

(e) to promote harmony and the spirit of common brotherhood amongst all the people of India transcending religious, linguistic and regional or sectional diversities; to renounce practices derogatory to the dignity of women;

(f) to value and preserve the rich heritage of our composite culture;

(g) to protect and improve the natural environment including forests, lakes, rivers, wild life and to have compassion for living creatures;

(h) to develop the scientific temper, humanism and the spirit of inquiry and reform;

(i) to safeguard public property and to abjure violence;

(j) to strive towards excellence in all spheres of individual and collective activity so that the nation constantly rises to higher levels of endeavour and achievements;

(k) who is a parent or guardian to provide opportunities for education to his child or, as the case may be, ward between age of six and fourteen years.

CHILDREN'S RIGHTS

Dear Children,

Wouldn't you like to know about your rights? Awareness about your rights will inspire and motivate you to ensure your protection and participation, thereby making social justice a reality. You may know that a commission for child rights is functioning in our state called the ***Kerala State Commission for Protection of Child Rights.***

Let's see what your rights are:

- Right to freedom of speech and expression.
- Right to life and liberty.
- Right to maximum survival and development.
- Right to be respected and accepted regardless of caste, creed and colour.
- Right to protection and care against physical, mental and sexual abuse.
- Right to participation.
- Protection from child labour and hazardous work.
- Protection against child marriage.
- Right to know one's culture and live accordingly.
- Protection against neglect.
- Right to free and compulsory education.
- Right to learn, rest and leisure.
- Right to parental and societal care, and protection.

Major Responsibilities

- Protect school and public facilities.
- Observe punctuality in learning and activities of the school.
- Accept and respect school authorities, teachers, parents and fellow students.
- Readiness to accept and respect others regardless of caste, creed or colour.

Contact Address:

Kerala State Commission for Protection of Child Rights
'Sree Ganesh', T. C. 14/2036, Vanross Junction
Kerala University P. O., Thiruvananthapuram - 34, Phone : 0471 - 2326603
Email: childrights.cpcr@kerala.gov.in, rte.cpcr@kerala.gov.in
Website : www.kescpcr.kerala.gov.in

Child Helpline - 1098, Crime Stopper - 1090, Nirbhaya - 1800 425 1400

Kerala Police Helpline - 0471 - 3243000/44000/45000

Online ***R. T. E Monitoring : www.nireekshana.org.in***